Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. The back of the first page and both sides of the last page are reserved for scratch-work.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	12	
4	14	
5	12	

Problem	Total Points	Score
6	12	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. *(2 points per part)* Each of the following multiple choice problems has one correct answer. Circle it. You do not need to show any reasoning.

(a) Suppose $|\mathbf{a} \times \mathbf{b}| > -\mathbf{a} \cdot \mathbf{b} > 0$. Then the angle between $\mathbf{a}$ and $\mathbf{b}$ is between...

(i) $0°$ and $45°$. (ii) $45°$ and $90°$. (iii) $90°$ and $135°$. (iv) $135°$ and $180°$.

(b) Suppose $\text{proj}_{\mathbf{a}}\mathbf{b} = \langle 1, -1, 1 \rangle$. Then $\mathbf{b}$ could be...

(i) $\langle 2, -2, 2 \rangle$. (ii) $\langle -1, 1, -1 \rangle$. (iii) $\langle 2, 2, 2 \rangle$. (iv) $\langle 2, 3, 4 \rangle$.

(c) The intersection of the hyperboloid $x^2 + y^2 - z^2 = 1$ and the xy-plane is...

(i) a line. (ii) a circle. (iii) a hyperbola. (iv) the empty set.

(d) The surface $z = f(x, y) = x^3 + y^3 - 3x - 3y$ has a local maximum of...

(i) $f(1, 1)$. (ii) $f(1, -1)$. (iii) $f(-1, 1)$. (iv) $f(-1, -1)$.

(e) A lamina occupies the disc $x^2 + y^2 \leq 1$, and the density at (x, y) is $\rho(x, y) = x^3 + y^2 + 2$. The center of mass of the lamina is...

(i) at the origin. (ii) on the x-axis. (iii) on the y-axis. (iv) none of these.

2. *(4 points per part)* For each part, consider the space curve of the vector function

$$\mathbf{r}(t) = \langle t^2 + 1, \cos(t) + 4t, 3t \rangle.$$

(a) Find parametric equations for the line tangent to the space curve at $t = 0$.

(b) Find the unit tangent vector to the space curve at $t = 0$.

(c) Find the curvature of the space curve at $t = 0$.

3. *(6 points per part)* For parts (a) and (b), let $\mathcal{S}$ be the implicitly defined surface

$$x\cos(z) + y^2 z - x^2 e^y + 20 = z.$$

(a) Find $\dfrac{\partial z}{\partial x}$ for points on $\mathcal{S}$.

(b) Find all intersections of $\mathcal{S}$ with the x-axis.

4. *(14 points)* Let $\mathcal{D}$ be the triangular region with vertices $(0,0)$, $(0,6)$, and $(3,0)$.

   Find the absolute maximum and minimum values of $f(x,y) = x^2 + xy - 2x$ on $\mathcal{D}$.

5. *(6 points per part)* **The two parts of this problem are unrelated.**

   (a) Evaluate the iterated integral

$$\int_0^2 \int_{\frac{y}{2}}^1 y\cos(x^3 - 1)dx\, dy.$$

   (b) Find the volume of the solid under the plane $3x + 2y - z = 0$ and above the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

6. *(12 points)* Set up and evaluate a double integral in polar coordinates to calculate the area of the region between the two polar curves $r = 6 + 2\sin(3\theta)$ and $r = 3 + 2\sin(3\theta)$.

7. *(7 points per part)* Let $f(x) = e^{2x} + \ln(1 - x) - x^2$.

(a) Find the second Taylor polynomial, $T_2(x)$, for $f(x)$ based at $b = 0$.

(b) Find (and justify) an error bound for $|f(x) - T_2(x)|$ on the interval $[-0.5, 0.5]$.

8. For this problem, you may use the following basic Taylor series:

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \qquad \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \qquad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k.$$

(a) *(6 points)* Find the Taylor series for $g(x) = \dfrac{1}{4+9x^2}$ based at $b = 0$.

Give your answer **using Σ-notation** and **list the first three nonzero terms**.

(b) *(5 points)* Find the Taylor series for $h(x) = \arctan\left(\dfrac{3x}{2}\right)$ based at $b = 0$.

Give your answer **using Σ-notation** and **list the first three nonzero terms**.

(c) *(3 points)* Find the open interval of convergence for the series you found in part (b).

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