CHECK that your exam contains 8 problems.

This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of hand-written notes and a TI-30X IIS calculator. Do not share notes or calculators.

Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)

In order to receive full credit, you must show all of your work.

Place a box around YOUR FINAL ANSWER to each question.

If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.

Raise your hand if you have a question.

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1. (12 points) In BOTH parts, consider the three points $A(1, 1, 2)$, $B(3, 1, 5)$, and $C(1, -2, 4)$.

(a) Find, to the nearest degree, the angle $\angle ABC$. (That is, find the angle at the corner $B$).

(b) Let $P$ be the plane that passes through $A$, $B$, and $C$.

Let $L$ be the line that passes through $(1, 0, 2)$ and $(6, 5, 7)$.

Find the $(x, y, z)$ point of intersection of the line $L$ and the plane $P$. 
2. (12 points) **NOTE: The two parts below are NOT related!**

(a) Find all values of $t$ at which the tangent line to the curve $x = 6 - t^3$, $y = 2t - 5t^2$ is parallel to the vector $\langle 3, 8 \rangle$.

(b) A small bug is moving according to the vector function $\mathbf{r}(t) = \langle t \sin(\pi t), \ln(t), t^2 - 4e^{2-2t} \rangle$. At time $t = 1$, the bug leaves the curve and follows the path of the tangent line. Find the $(x, y, z)$ coordinates where the bug’s tangent line path would intersect the $xy$-plane.
3. (12 points) Consider the polar curve given by the polar function

\[ r = 2 + \sin(\theta). \]

Let \( P \) be the point of intersection of the polar curve with the line \( y = \frac{\sqrt{3}}{3}x \) in the first quadrant.

(a) Find an equation of the tangent line to the curve at \( P \).

(b) Find the area of the region bounded by the polar curve, the line \( y = \frac{\sqrt{3}}{3}x \) and the \( x \)-axis in the first quadrant.
4. (14 points) **NOTE: The parts below are NOT related!**

(a) Compute the partial derivatives with respect to $x$ and $y$ of the function $f(x, y) = x^y$.

(b) Calculate the tangent plane to the surface defined by the equation

$$xy \sin(z) + x - y + z = 0$$

at the point $(x, y, z) = (2, 2, 0)$.

(c) Find an example of a differentiable function $f(x, y)$ whose best linear approximation near $(0, 0)$ is given by the function $L(x, y) = 0$ and which has neither a local maximum nor a local minimum at $(0, 0)$. 
5. (12 points) NOTE: The parts below are NOT related!

(a) Find the volume of the solid above the cone \( z = \sqrt{x^2 + y^2} \) and below the sphere \( x^2 + y^2 + z^2 = 18 \).

(b) Evaluate the integral
\[
\int_0^1 \int_x^1 \sqrt{1+y^2} \, dy \, dx.
\]
6. (12 points) The acceleration vector of a spaceship is

\[ \vec{a}(t) = \langle -2 \cos(t) - 3 \sin(t), -\cos(t) + 6 \sin(t), \sqrt{40} \cos(t) \rangle \quad \text{for all } t \geq 0 \]

and the specified initial velocity and position are

\[ \vec{v}(0) = \langle 3, -6, 0 \rangle \quad \vec{r}(0) = \langle 2, 1, 0 \rangle. \]

(a) Find the position function \( \vec{r}(t) \) of the spaceship.

(b) Find the normal component of the acceleration at time \( t \). (Hint: The answer is a constant, simplify until you find this constant.)
7. (12 points) Consider the function $f(x) = \ln(1 + 3x) + xe^{-2x} - \frac{4x}{1 + 5x}$.

(a) Find the Taylor series for $f(x)$ based at $b = 0$. Write your answer using sigma $\Sigma$ notation.

(b) Find the open interval on which the series in (a) converges.

(c) Find the third Taylor polynomial of $F(x)$ based at $b = 0$ where

$$F(x) = \int_0^x f(t) \, dt.$$
8. (14 points) Consider the function \( f(x) = \sin(\pi x) + \cos(\pi x) + \frac{1}{2-x} \).

(a) Find the second Taylor polynomial \( T_2(x) \) for \( f(x) \) based at \( b = 1 \).

(b) Find an upper bound on the error \( |T_2(x) - f(x)| \) on the interval \( \left[ \frac{1}{2}, \frac{3}{2} \right] \).

(c) Find a smaller interval \( I \) centered at \( b = 1 \) so that the error \( |T_2(x) - f(x)| \) has an upper bound 0.001 for all \( x \) in the interval \( I \).