Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	15	
3	9	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	12	
8	12	
9	12	
Total	100	

1. (10 points) Consider the four points $(2, 2, 2), (1, 0, 0), (2, 3, 7)$, and $(5, 5, 5)$.

   (a) Do all four points lie in a common plane? Justify your answer.

   (b) Let $\mathbf{v} = \langle 4, 5, 6 \rangle$ and $\mathbf{n}$ be a vector orthogonal to the plane containing the points $(2, 2, 2)$, $(1, 0, 0)$, and $(2, 3, 7)$. Find the vector projection of $\mathbf{v}$ along $\mathbf{n}$.

2. (15 points) Consider the vector function $\mathbf{r}(t) = \left\langle 2t, \frac{1}{2}t^2, \frac{4}{3}t^{3/2} \right\rangle$, for $t \geq 0$.

   (a) Calculate the unit tangent vector $\mathbf{T}(t)$.

   (b) Give the equation of the normal plane at $t = 1$.

   (c) Compute the curvature $\kappa(t)$ and $\lim_{t\to\infty} \kappa(t)$. Explain briefly what this limit means about the curve.

3. (9 points) Let X be the surface in $\mathbb{R}^3$ determined by the equation $z^2 = x^2 - 2y^2$. **You are not required to show any work for the following questions.**

(a) Identify the traces of X in the indicated plane.

i. The trace in the plane $x = 0$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

ii. The trace in the plane $x = k$ $(k \neq 0)$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

iii. The trace in the plane $y = 0$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

iv. The trace in the plane $y = k$ $(k \neq 0)$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

v. The trace in the plane $z = 0$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

vi. The trace in the plane $z = k$ $(k \neq 0)$ is a(n):

______	circle	______	ellipse
______	hyperbola	______	parabola
______	point	______	pair of lines

(b) Identify the surface X.

______	cone	______	ellipsoid
______	elliptic paraboloid	______	hyperbolic paraboloid
______	hyperboloid of one sheet	______	hyperboloid of two sheets

(c) True or False?

______ The path described by the vector function $\mathbf{r}(t) = \left\langle t, \frac{1}{2}t, \frac{1}{\sqrt{2}}t \right\rangle$ lies on X.

______ Every line that lies on X intersects the path described by $\mathbf{r}(t) = \left\langle t, \frac{1}{2}t, \frac{1}{\sqrt{2}}t \right\rangle$.

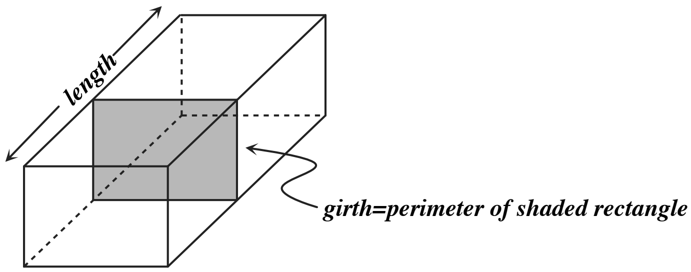
4. (10 points) Find an equation of the plane that contains the line of intersection of the planes $x - z = 1$ and $y + 2z = 2$ and is perpendicular to the plane $x + y - 2z = 3$.

5. (10 points) Let $f(x, y) = y^3 \cos(x - y) + xe^{4-xy} - 16$.

   (a) Find the tangent plane to the surface at the point $(2, 2)$.

   (b) Use part (a) to approximate $f(2.01, 1.95)$.

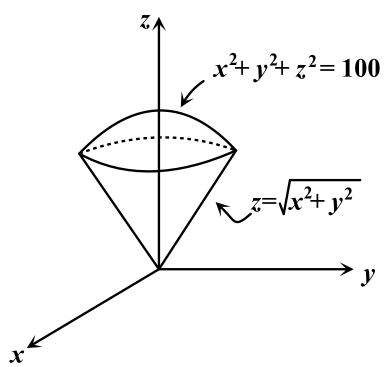
6. (10 points) A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108 in. Find the dimensions of the package with largest volume that can be mailed. (For full credit, you must indicate how you know that these dimensions give the **maximum** volume.)



7. (12 points)

   (a) Compute $\int_0^1 \int_{\sqrt{y}}^1 \cos x^3 \, dx \, dy$.

   (b) Compute the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 100$ and below by the cone $z = \sqrt{x^2 + y^2}$.



8. (12 points) Let $f(x) = x^3 e^{x^2}$.

   (a) Find the Taylor series for $f(x)$ based at $b = 0$. Write the series using a single Σ sign.

   (b) Give the fifth Taylor polynomial for $f(x)$ based at $b = 0$, $T_5(x)$.

   (c) Find the Taylor series based at $b = 0$ for

   $$g(x) = \int_0^x f(t)\, dt.$$

   Write the series using a single Σ sign.

9. (12 points) Let $h(x) = x^{1/3}$.

   (a) Find $T_2(x)$, the second Taylor polynomial for $h(x)$ based at $b = 8$.

   (b) Use the Quadratic Approximation Error Bound to find an upper bound for the error $|f(x) - T_2(x)|$ on the interval $[7, 9]$.

   (c) Use your answer to part (a) to approximate $\sqrt[3]{9}$.