Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	12	
4	12	
5	6	

Problem	Total Points	Score
6	12	
7	12	
8	12	
9	12	
Total	100	

1. (10 points) Indicate whether each of the following is True or False. You do not need to explain your answer.

   (a) ______ A vector is an object with two properties: length and direction.

   (b) ______ The planes $x + y + z = 0$ and $x + y - z = 1$ are parallel.

   (c) ______ If $\mathbf{r}(t)$ is a vector-valued function such that $|\mathbf{r}'(t)| = 1$, then the unit normal vector $\mathbf{N}(t)$ is parallel to $\mathbf{r}''(t)$.

   (d) ______ The function $f(x, y) = \sin(x)\sin(y)$ has infinitely many saddle points.

   (e) ______ If $f(x)$ is a polynomial of degree 2, then the second Taylor polynomial for $f(x)$ based at b is equal to $f(x)$ for any b.

   (f) ______ If the linearization of a differentiable function $z = f(x, y)$ at the point $(1, -1, 7)$ is $L(x, y) = 3x - 4y$, then $f_x(1, -1) = 3$ and $f_y(1, -1) = 4$.

   (g) ______ Let $f(x, y) = x^2 - y^2 - 2x + 2y$. Then $f(x, y)$ has a local maximum at $(1, 1)$.

   (h) ______ Let $f(x, y)$ be a continuous function. Then

   $$\int_0^{\frac{\pi}{2}} \int_0^{\sin x} f(x, y)\, dy\, dx = \int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} f(x, y)\, dx\, dy.$$

   (i) ______ If $f(x, y)$ has two local maxima, then f must have a local minimum.

   (j) ______

   $$\int_0^1 \int_0^2 \sin(x^3 + y^3) y\, dx\, dy \leq 2.$$

2. (12 points) Consider the points $(1,2,3),(2,3,4),(1,1,1),(1,1,2)$.

   (a) Do the four points lie in a common plane? Show work to justify your answer.

   (b) Find an equation for the plane that contains the point $(0,0,-1)$ and is parallel to the plane that contains the points $(2,3,4),(1,1,1),(1,1,2)$.

3. (12 points) A particle moves along a path corresponding to a vector-valued position function $\mathbf{r}(t)$ with velocity $\mathbf{r}'(t) = \langle \cos(t), -\sin(t), -2\sin(t) \rangle$.

   (a) Find the curvature of the path as a function of t.

   (b) Find the position vector $\mathbf{r}(t)$ if the path lies on the paraboloid $z = x^2 + y^2$.

4. (12 points) Classify all critical point(s) of the function $f(x, y) = x^y - xy$ in the domain $x > 0, y > 0$.

5. (6 points) Describe in parametric form the line through $(1, 1, -1)$ that is parallel to the normal vector of the plane $4x + 5y + 6z = z - 7$.

6. (12 points) Find the shortest distance between the curve

$$\{(x, y, 4) \mid y = 4 - x^2\}$$

and the curve

$$\{(0, y, z) \mid z = \sqrt{y^2 + 4} + 4\}.$$

7. (12 points) Use a double integral to find the area of the region that is enclosed by the curve $r = 6 + 4\cos(2\theta)$.

8. (12 points) Let $f(x) = x^2 - 3x + 2x \ln x$.

   (a) Find the third Taylor polynomial $T_3(x)$ for $f(x)$ based at $b = 1$.

   (b) Let a be a real number such that $0 < a \leq \frac{1}{2}$ and let J be the closed interval $[1-a, 1+a]$. Use Taylor's inequality to find an upper bound for the error $|f(x) - T_3(x)|$ on the interval J.

   (c) Find a value of a such that $|f(x) - T_3(x)| \leq 0.001$ for all x in $J = [1-a, 1+a]$.

9. (12 points) Let
$$g(x) = \frac{1}{1+x^2} + e^{-2x^2}.$$

(a) Find the Taylor series for $g(x)$ based at 0. Write the series using one Σ sign.

(b) Find the interval on which the series in (a) converges.

(c) Use the first three nonzero terms of the Taylor series in part (a), to approximate the value of the integral
$$\int_0^1 \frac{1}{1+x^2} + e^{-2x^2}\, dx.$$
(Give your final answer to four digits after the decimal point).