MATH 126 – FINAL EXAM Hints and Answers
WINTER 2011

1. HINT: The curves intersect when $t = 1$ and $s = 2$ and the tangent vectors at the point of intersection are $\mathbf{r}'(1) = \langle 1, 2, 3\rangle$ and $\mathbf{r}'(s) = \langle 1, 5, 8\rangle$.

   ANSWER: $\theta = \cos^{-1}\left(\dfrac{\sqrt{35}}{6}\right)$

2. (a) ANSWER: $\mathbf{T}(t) = \left\langle -\frac{5}{13}\sin t, -\frac{12}{13}\sin t, \cos t\right\rangle$
   (b) ANSWER: $\mathbf{N}(t) = \left\langle -\frac{5}{13}\cos t, -\frac{12}{13}\cos t, -\sin t\right\rangle$
   (c) ANSWER: $\mathbf{B}(t) = \left\langle \frac{12}{13}, -\frac{5}{13}, 0\right\rangle$
   (d) ANSWER: $\kappa(t) = \dfrac{1}{13}$

3. (a) F; (b) F; (c) F; (d) T; (e) T; (f) T; (g) T; (h) F; (i) F; (j) F

4. (a) ANSWER: (answers may vary) ℓ_1: $x = 4t,\ y = 3 - 2t,\ z = 4$
   (b) ANSWER: (answers may vary) $\ell_2 : x = 3 + 2t,\ y = -1 - t,\ z = 5$.
   To verify that the two lines are parallel, note that the direction vector for ℓ_1 is $\vec{v}_1 = \langle 4, -2, 0\rangle$ and the direction vector for ℓ_2 is $\vec{v}_2 = \langle 2, -1, 0\rangle$ and $\vec{v}_1 = 2\vec{v}_2$.
   (c) ANSWER: $x + 2y + 5z = 26$

5. (a) HINT: Compute $f_x(100, 100)$.
   ANSWER: You will be ascending at an angle of $\tan^{-1}\left(\frac{1}{10}\right)$ from the horizontal.
   (b) HINT: Compute $f_y(100, 100)$.
   ANSWER: You will be descending at an angle of $\tan^{-1}\left(\frac{3}{10}\right)$ from the horizontal. (Would also accept $\tan^{-1}\left(-\frac{3}{10}\right)$.)
   (c) ANSWER: $z = 990 + \frac{1}{10}(x - 100) - \frac{3}{10}(y - 100)$
   (d) ANSWER: $f(100, 102) \approx 990 + \frac{1}{10}(100 - 100) - \frac{3}{10}(102 - 100) = 989.4$ feet

6. HINT: Let $P(x, y, z)$ be a point on the hyperbolololoid. Then the coordinates of P satisfy the equation $z^2 = 1 + 2x^2 + y^2$. The distance from P to Q is then

$$\sqrt{(x-0)^2 + (y-1)^2 + (z-0)^2} = \sqrt{x^2 + (y-1)^2 + z^2} = \sqrt{x^2 + (y-1)^2 + (1 + 2x^2 + y^2)}.$$

   The distance will be smallest when the expression under the square root is minimized. So, let

$$f(x, y) = x^2 + (y-1)^2 + (1 + 2x^2 + y^2),$$

   (the square of the distance from P to Q) and find the point (x, y) that minimizes f.

   ANSWER: $(0, \frac{1}{2}, \pm\frac{\sqrt{5}}{2})$

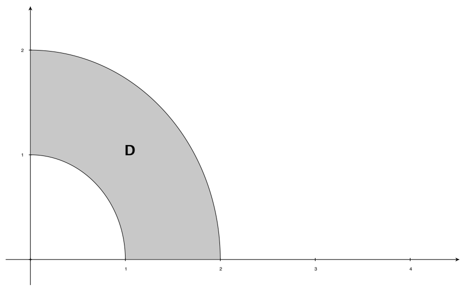
7. (a) ANSWER: $T_2(x) = 1 + 3(x - 1) + \frac{7}{2}(x - 1)^2$
   (b) ANSWER: $f(0.9) \approx T_2(0.9) = 1 + 3(-0.1) + \frac{7}{2}(-0.1)^2 = 0.735$
   (c) HINT: $f'''(x) = e^{x-1}(x^2 + 6x + 6)$, which is increasing on the interval $I = [0.9, 1]$. So, for all x in I, $|f'''(x)| \leq f'''(1) = 13$.
   ANSWER: $|f(x) - T_2(x)| \leq \dfrac{13}{3!}|0.9 - 1|^3 \approx 0.0022$ (would accept a larger bound)

8. (a) ANSWER: $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(2k+1)} x^{2k+1}$

   (b) ANSWER: $x - \frac{1}{3!3}x^3 + \frac{1}{5!5}x^5 - ...$

   (c) ANSWER: $(-\infty, \infty)$

9. HINT: The region D is:



D is a polar rectangle: $\{(r,\theta) : 1 \le r \le 2, 0 \le \theta \le \frac{\pi}{2}\}$.

The area of D is $\frac{3\pi}{4}$.

Then, the average value of f over D is:

$$\frac{1}{\text{area of } D} \iint_D e^{-(x^2+y^2)}\, dA = \frac{4}{3\pi} \int_0^{\pi/2} \int_1^2 e^{-r^2} r\, dr\, d\theta.$$

ANSWER: $f_{ave} = \frac{e^{-1} - e^{-4}}{3}$