Math 126	Final Examination	Spring 2024
Your Name	Your Signature	
Student ID #		Quiz Section
Professor's Name	TA's Name	

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$ " × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.

• Place a box around **YOUR FINAL ANSWER** to each question.

- If you need more room, use the back of the first page or either side of the last page and indicate that you have done so. If you *still* need more room, ask for more scratch paper.
- Do not write within 1 centimeter of the edge of the page.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	12	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. (12 points) Find the equation for the plane that passes through the point (1, 2, 4) and contains the line given by x = 2t, y = 3 - t, z = 2 + 5t. And give the x-intercept of this plane.

Plane Equation: _____

x-intercept: (x, y, z) =

2. (12 points) Let a curve γ be the intersection of the plane y+z = 1 and the cylinder $x^2+y^2 = 1$. Find the points where γ has maximum curvature. 3. (6 points per part) Parts (a) and (b), consider the surface implicitly defined by the equation

$$z(\cos(z) + 2) = x^2 - y^2.$$

(a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

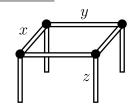


(b) Give the linear approximation, i.e. tangent plane, to the surface at the point where (x, y) = (1, 1). And use this approximation to estimate the z-coordinate of a point on the surface where (x, y) = (1.01, 0.98).

Linearization: $_$

4. (12 points) Find the dimensions of a rectangular tent of maximum volume such that the sum of the lengths of its 8 posts (above the ground) is 36 m.

To receive full credit, you must justify that your answer is indeed the maximum.



5. (6 points per part) For the following problems, set up an iterated integral as described.

You do not have to evaluate the integrals. Just set them up.

(a) Reverse the order of integration of
$$\int_0^2 \int_{2^{(3-x)}}^{8-3x} \sin(x^2) \, dy \, dx.$$

Integral: _____

(b) Write a double integral for the volume below the cone $z = \sqrt{x^2 + y^2}$ and above the paraboloid $z = x^2 - x + y^2$.

(Hint: convert to polar!)

6. (12 points) Let \mathcal{L} be the lamina whose shape is the triangular region with vertices (0,0), (0,2), and (1,2), and whose density at any point is proportional to its distance to the y-axis.

Find the center of mass of \mathcal{L} .

- 7. For this problem, let $g(x) = (3+x)^{3/2}$ and consider the Taylor polynomials based at b = 1.
 - (a) (5 points) Find the second Taylor polynomial, $T_2(x)$, for the function g based at b = 1.

 $T_2(x) =$ _____

(b) (4 points) Use $T_2(x)$ to approximate the value of $\sqrt{4.4^3}$

 $\sqrt{4.4^3} \approx _$

(c) (5 points) Use Taylor's inequality to find an upper bound for the error in your approximation in part (b).

Upper bound: _____

- 8. For this problem, let $f(x) = \frac{16}{4 x^2} + 6\cos(2x)$
 - (a) (6 points) Give the Taylor series for f based at b = 0 using one sigma sign.

Taylor series: _

(b) (3 points) Find the largest open interval on which this Taylor series converges.

interval: _____

(c) (5 points) Find $T_3(x)$, the third Taylor polynomial for f based at b = 0, then estimate the value of $\int_0^1 f(x) dx$ by replacing f(x) with $T_3(x)$.

 $\int_{0}^{1} f(x) \, dx \approx \underline{\qquad}$

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see last page" below a problem.

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see last page" below a problem.