Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	12	
7	14	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle “see first page” below a problem.

1. *(4 points per part)* **Parts (a), (b), and (c) are unrelated.**

   (a) The two lines $x = 1 + t,\ y = t,\ z = 1 - 2t$ and $x = 1,\ y = 4t,\ z = 1 - 3t$ intersect at the point $(1, 0, 1)$. Find the equation for the plane containing these two intersecting lines.

   equation for plane: ____________________

   (b) Find parameteric equations for the line of intersection of the two planes $x + y + z = 1$ and $2x + y - 3z = -3$.

   equations for line: ____________________

   (c) Find all points of intersection of the line through $(0, 0, 1)$ and $(3, 4, 1)$ and the paraboloid $100z = x^2 + y^2$.

   intersection point(s): $(x, y, z) =$ ____________________

2. *(6 points per part)* **Parts (a) and (b) are unrelated.**

   (a) Find a vector function for the curve of intersection between the surface $5x^2 + y^2 - z^2 = 4$ and the plane $z = x$.

   vector function: ________________________

   (b) Find the curvature of $\mathbf{r}(t) = \langle t^3, t^2 - 1, 3t + 7 \rangle$ at the point $(-8, 3, 1)$.

   curvature: ________________

3. *(6 points per part)* For parts (a) and (b), let $f(x, y) = x^2y^2 - 4x^2 - y^2$.

   (a) Find all the saddle points of $f(x, y)$.

   saddle points: ______________________

   (b) Find the equation of the tangent plane to the surface $z = f(x, y)$ at $x = 2, y = 1$ and use it to approximate $f(2.1, 0.8)$.

   tangent plane: ______________________

4. *(12 points)* Find the absolute maximum and absolute minimum values of the function

$$f(x, y) = x^2 - y^2 + 2y$$

on the region enclosed by $x = 0$, $y = 4$ and $y = x^2$.

absolute extrema: ______________________________

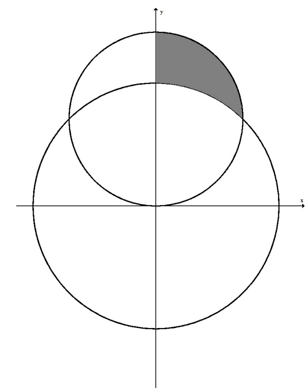
5. *(12 points)* Reverse the order of integration and evaluate

$$\int_0^2 \int_{x/2}^1 \frac{x}{y^3+1}\, dydx.$$

answer = ____________________

6. *(12 points)* Let R be the region inside the circle $x^2 + y^2 = 4y$, outside the circle $x^2 + y^2 = 8$, and in the first quadrant (shown below). Evaluate

$$\iint_R \frac{x}{x^2 + y^2}\, dA$$



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answer = ____________________

7. For parts (a)–(c), let $f(x) = \ln(2x - 1)$.

(a) *(5 points)* Find the second Taylor polynomial, $T_2(x)$, for $f(x)$ based at $b = 1$.

$T_2(x) =$ ____________________

(b) *(4 points)* Use your answer to part (a) to approximate $\ln(1.1)$.

$\ln(1.1) \approx$ ____________

(c) *(5 points)* Find an upper bound (as sharp as possible) on the error for your answer from part (b).

Error bound: ____________

8. For this problem, you may use the following basic Taylor series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(a) *(6 points)* Find the Taylor series for $f(x) = \int_0^x e^{2t^2}\, dt$ based at $b = 0$. Express your answer using $\sum$–notation.

Taylor series: ____________________________

(b) *(3 points)* Find the open interval of convergence for the series you found in (a).

Interval of convergence: ____________________

(c) *(5 points)* Find $f^{(2023)}(0)$, i.e. the 2023$^{\text{rd}}$ derivative of f at 0.

$f^{(2023)}(0) =$ ____________________

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