• CHECK that your exam contains 8 problems on 8 pages.

• This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.

• Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)

• In order to receive full credit, you must show all of your work.

• Place a box around YOUR FINAL ANSWER to each question.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.

• Raise your hand if you have a question.
1. (10 points) Each of the following multiple choice problems has one correct answer. Circle it. You do not need to show any reasoning.

(a) Suppose $\mathbf{c} = \mathbf{a} - \frac{1}{2} |\mathbf{b}|$. Then the angle between $\mathbf{a}$ and $\mathbf{b}$ is...

(i) $30^\circ$.  
(ii) $45^\circ$.  
(iii) $60^\circ$.  
(iv) $90^\circ$.

(b) Suppose $\mathcal{S}$ is the set of points $P$ such that the distance from $P$ to the $x$-axis is equal to 3. Then $\mathcal{S}$ is...

(i) a plane.  
(ii) a cylinder.  
(iii) a sphere.  
(iv) a cone.

(c) The surface $z = x^2 + 2xy$ is tangent to the plane $z = 6x + 4y - 8$ at the point...

(i) $(-2, 3, -8)$  
(ii) $(0, 2, 0)$.  
(iii) $(2, 1, 8)$.  
(iv) $(4, 0, 16)$.

(d) The value of $\int_2^5 \int_3^5 (5 + \sin^2(yx^2 + y^3)) \, dy \, dx$ is between...

(i) 0 and 10.  
(ii) 10 and 20.  
(iii) 20 and 30.  
(iv) 30 and 40.

(e) The Taylor series for $f(x) = \frac{1}{2 - x^2}$ centered at $b = 0$ converges on the interval...

(i) $(-1, 1)$.  
(ii) $(-2, 2)$.  
(iii) $(-4, 4)$.  
(iv) $(-\sqrt{2}, \sqrt{2})$. 
2. (12 pts) Let $L$ be the line of intersection of the two planes

\[ x + y + 2z = c \quad \text{and} \quad x - cy - cz = -1 \]

where $c$ is some real number. Find a value of $c$ for which $L$ is perpendicular to the plane $3x - y - z = 0$. 
3. (12 pts) Find the curvature of the ellipse

\[ x = 3 \cos(t), \quad y = 4 \sin(t), \quad z = 1, \]

at the points \((3, 0, 1)\) and \((0, 4, 1)\).
4. (14 pts) Find and classify all the critical points of \( f(x, y) = 4xy - 3y + \frac{1}{x} - \frac{1}{4} \ln(y) \).

Clearly show your work in using the second derivative test and label your answers.
5. (14 pts) Compute the volume of the solid between the surface $x^2 + y + z = 4$ and the $xy$-plane above the first quadrant.
6. (12 pts) Compute
\[ \iint_R e^{-(x^2+y^2)} \, dA \]
where \( R = \{(x, y) : x^2 + y^2 \leq 9\} \).
7. (12 pts) Let \( f(x) = 1 + x + x^2 + 3x^3 \).

(a) Find the second-degree Taylor polynomial, \( T_2(x) \), for \( f(x) \) based at \( b = 1 \).

(b) Determine an interval around \( b = 1 \) on which

\[ |T_2(x) - f(x)| < 0.024. \]
8. (14 pts) Let \( f(x) = \frac{x^3}{1 + x^4} \).

(a) Find the Taylor series for \( f(x) \) based at zero. Express your answer using sigma notation.

(b) Use the Taylor series you found in (a) to find the Taylor series for

\[ g(x) = x^2 \ln(1 + x^4). \]

Express your answer using sigma notation.