Math 126 Spring 2017
Final Exam Answers

1. $(x, y, z) = \left(1, \frac{1}{3}, \frac{10}{3}\right)$

2. $(x, y, z) = \left(2e^{-2} + 2, 6, 0\right)$

3. (a) at $(1, 2, 1)$, $\dfrac{\partial z}{\partial x} = -4$ and $\dfrac{\partial z}{\partial y} = -1$

   (b) $f(1.01, 1.99) \approx -4(1.01 - 1) - (1.99 - 2) + 1 = 0.97$

4. $(2, -2, 1 + \sqrt{13})$ and $(2, -2, 1 - \sqrt{13})$

5. $\dfrac{9\pi}{2}$

6. $-\dfrac{128}{3}$

7. $\dfrac{1}{4} - \dfrac{1}{4}\cos(16)$

8. (a) $g(x) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k 4^k x^k}{3^{k+1}} = \frac{1}{3} - \frac{4x}{3^2} + \frac{4^2x^2}{3^3} + ...$

   (b) $h(x) = \ln(3) + \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k 4^{k+1} x^{k+1}}{3^{k+1}(k+1)} = \ln(3) + \frac{4x}{3} - \frac{4^2x^2}{3^2(2)} + \frac{4^3x^3}{3^3(3)} - ...$

   (c) $\left(-\dfrac{3}{4}, \dfrac{3}{4}\right)$

9. For $k = 0, 1, 2, ...,$ $f^{(k)}(x) = 3^k e^{3x}$. So, for all t in the interval $\left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$,

$$|f^{(n+1)}(t)| = |3^{n+1}e^{3t}| \leq 3^{n+1}e < 3^{n+2}.$$

   This means that, for all x in the interval $\left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$,

$$|f(x) - T_n(x)| \leq \frac{3^{n+2}}{(n+1)!}|x|^{n+1} \leq \frac{3^{n+1}}{(n+1)!} \cdot \frac{1}{3^{n+1}} = \frac{3}{(n+1)!}.$$

   We therefore need n so that $\dfrac{3}{(n+1)!} < \dfrac{1}{100}$.

   $n = 5$ is sufficient.