Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 9 problems.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a scientific calculator with no graphing, programming, or calculus capabilities. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	10	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	8	
7	10	
8	12	
9	12	
Total	100	

1. (12 points) Let ℓ be the line through $(8, -4, 7)$ and $(20, 2, 3)$.
   Let P be the plane through $(5, 3, 0)$, $(4, -1, 6)$, and $(6, 3, 1)$.

   (a) Where do the line ℓ and the plane P intersect?

   (b) Let θ be the angle between the line ℓ and a normal vector to the plane P.
   Find $|\cos\theta|$.

2. (10 points) Identify each statement as true or false. You do not need to offer any explanation.

   (a) **T** **F** For any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$, it is always true that $\text{comp}_{\mathbf{a}}\mathbf{b} \leq |\mathbf{b}|$.

   (b) **T** **F** The traces of the surface $3x - y^2 + 2z^2 = 1$ parallel to the yz-plane are parabolas.

   (c) **T** **F** If $\mathbf{a} = \langle 1, -1, 5\rangle$ and $\mathbf{b} = \langle 2, 4, 0\rangle$, then $\mathbf{a} \times \mathbf{b} = \langle 20, -10, -6\rangle$.

   (d) **T** **F** If $\mathbf{a}$ and $\mathbf{b}$ are non-zero and $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{a}|$, then $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

   (e) **T** **F** For any two vectors $\mathbf{a}$ and $\mathbf{b}$, it is always true that $|\mathbf{a} + \mathbf{b}| \geq |\mathbf{a}|$.

3. (12 points) Consider the polar curve (ellipse) $r = \dfrac{3}{2 + \cos\theta}$.

   (a) Find the equation in xy-coordinates of the line tangent to the curve at $\theta = \dfrac{\pi}{2}$.

   (b) Find all points (x, y) on the curve at which the tangent line is vertical.

4. (12 points) A particle is moving along a helix $\mathbf{r}(t) = \langle \sin t, \cos t, 2t \rangle$.

   (a) Find the distance traveled by the particle from $t = 0$ to $t = 3$.

   (b) Find an equation for the osculating plane at $t = 0$.

   (c) Find the curvature of this helix.

   (d) Find the tangential component (speed-changing part) and normal component (direction-changing part) of the acceleration.

5. (12 points)

   (a) Let $D = \{(x,y) : 1 \le x^2 + y^2 \le 4, x \ge 0\}$. Calculate the integral

$$\iint_D x \sin\left((x^2+y^2)^{\frac{3}{2}}\right) \cos\left((x^2+y^2)^{\frac{3}{2}}\right) dA.$$

   Give an exact answer.

   (b) Evaluate the integral

$$\int_0^1 \int_{\sqrt[3]{x}}^1 \sin(x/y)\, dy\, dx$$

6. (8 points) Find the tangent plane to the surface defined by the equation

$$x^3 + y^3 + z^3 + xyz = 0$$

at the point $(1, 0, -1)$.

7. (10 points) Let S be the surface defined by the equation

$$xyz = 2.$$

Find all points that lie on the surface S that yield a local minimum of the function

$$f(x, y, z) = x^2 + y^2 + 2z^2.$$

8. (12 points) Let $F(x) = \displaystyle\int_0^x \frac{t}{8+t^3}\,\mathrm{d}t$.

   (a) Find the Taylor series for the function $F(x)$ based at $b = 0$.

   (b) Find the open interval on which the series in part (a) converges.

   (c) Find the value of $F^{(11)}(0)$ (the eleventh derivative of F at 0). Give an exact answer.

9. (12 points) Consider the function $f(x) = x\ln(x-2)$.

   (a) Find the second Taylor polynomial $T_2(x)$ based at $b = 3$.

   (b) Use the second Taylor polynomial to approximate $f(3.1)$.

   (c) Use Taylor's inequality to find an upper bound for the error of your approximation is part (b).