• CHECK that your exam contains 9 problems.

• This exam is closed book. You may use one 8$\frac{1}{2}$ × 11 sheet of notes and a scientific calculator with no graphing, programming, or calculus capabilities. Do not share notes or calculators.

• Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)

• In order to receive full credit, you must show all of your work.

• Place a box around YOUR FINAL ANSWER to each question.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.

• Raise your hand if you have a question.
1. (12 points) Let \( \ell \) be the line through \((8, -4, 7)\) and \((20, 2, 3)\).
Let \( P \) be the plane through \((5, 3, 0)\), \((4, -1, 6)\), and \((6, 3, 1)\).

(a) Where do the line \( \ell \) and the plane \( P \) intersect?

(b) Let \( \theta \) be the angle between the line \( \ell \) and a normal vector to the plane \( P \).
Find \( |\cos \theta| \).
2. (10 points) Identify each statement as true or false. You do not need to offer any explanation.

(a) T  F  For any two non-zero vectors \( \mathbf{a} \) and \( \mathbf{b} \), it is always true that \( \text{comp}_a \mathbf{b} \leq |\mathbf{b}| \).

(b) T  F  The traces of the surface \( 3x - y^2 + 2z^2 = 1 \) parallel to the \( yz \)-plane are parabolas.

(c) T  F  If \( \mathbf{a} = \langle 1, -1, 5 \rangle \) and \( \mathbf{b} = \langle 2, 4, 0 \rangle \), then \( \mathbf{a} \times \mathbf{b} = \langle 20, -10, -6 \rangle \).

(d) T  F  If \( \mathbf{a} \) and \( \mathbf{b} \) are non-zero and \( \mathbf{a} \cdot \mathbf{b} = |\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{a}| \), then \( \mathbf{a} \) and \( \mathbf{b} \) are orthogonal.

(e) T  F  For any two vectors \( \mathbf{a} \) and \( \mathbf{b} \), it is always true that \( |\mathbf{a} + \mathbf{b}| \geq |\mathbf{a}| \).
3. (12 points) Consider the polar curve (ellipse) \( r = \frac{3}{2 + \cos \theta} \).

(a) Find the equation in \( xy \)-coordinates of the line tangent to the curve at \( \theta = \frac{\pi}{2} \).

(b) Find all points \((x, y)\) on the curve at which the tangent line is vertical.
4. (12 points) A particle is moving along a helix \( \mathbf{r}(t) = (\sin t, \cos t, 2t) \).

(a) Find the distance traveled by the particle from \( t = 0 \) to \( t = 3 \).

(b) Find an equation for the osculating plane at \( t = 0 \).

(c) Find the curvature of this helix.

(d) Find the tangential component (speed-changing part) and normal component (direction-changing part) of the acceleration.
5. (12 points)

(a) Let \( D = \{ (x, y) : 1 \leq x^2 + y^2 \leq 4, x \geq 0 \} \). Calculate the integral

\[
\iint_D x \sin \left( (x^2 + y^2)^{\frac{3}{2}} \right) \cos \left( (x^2 + y^2)^{\frac{3}{2}} \right) \, dA.
\]

Give an exact answer.

(b) Evaluate the integral

\[
\int_0^1 \int_{\sqrt{x}}^1 \sin(x/y) \, dy \, dx
\]
6. (8 points) Find the tangent plane to the surface defined by the equation

\[ x^3 + y^3 + z^3 + xyz = 0 \]

at the point \((1, 0, -1)\).
7. (10 points) Let $S$ be the surface defined by the equation

$$xyz = 2.$$ 

Find all points that lie on the surface $S$ that yield a local minimum of the function

$$f(x, y, z) = x^2 + y^2 + 2z^2.$$
8. (12 points) Let \( F(x) = \int_0^x \frac{t}{8 + t^3} \, dt \).

(a) Find the Taylor series for the function \( F(x) \) based at \( b = 0 \).

(b) Find the open interval on which the series in part (a) converges.

(c) Find the value of \( F^{(11)}(0) \) (the eleventh derivative of \( F \) at \( 0 \)). Give an exact answer.
9. (12 points) Consider the function $f(x) = x \ln(x - 2)$.

(a) Find the second Taylor polynomial $T_2(x)$ based at $b = 3$.

(b) Use the second Taylor polynomial to approximate $f(3.1)$.

(c) Use Taylor’s inequality to find an upper bound for the error of your approximation is part (b).