Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a scientific calculator with no graphing, programming, or calculus capabilities. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	10	
4	10	
5	8	

Problem	Total Points	Score
6	12	
7	12	
8	12	
9	12	
Total	100	

1. (12 points) Consider $f(x) = 5\ln(x) - x$.

   (a) Find the second Taylor polynomial, $T_2(x)$, for $f(x)$ based at $b = 1$.

   (b) Let a be a real number such that $0 < a < 1$ and let J be the closed interval $[1-a, 1+a]$. Use the Quadratic Approximation Error Bound to find an upper bound for the error $|f(x) - T_2(x)|$ on the interval J. (Your answer will be in terms of a).

   (c) Find a value of a such that $0 < a < 1$ and $|f(x)-T_2(x)| \leq 0.36$ for all x in $J = [1-a, 1+a]$.

2. (12 points) Consider $g(x) = xe^{x^3} + \dfrac{2x}{2 - x^3}$.

   (a) Find the Taylor series for $g(x)$ based at $b = 0$. Write the series using one Σ sign.

   (b) Find the open interval on which the series in (a) converges.

   (c) Use the first three nonzero terms of the Taylor series in part (a) to approximate the value of the integral

$$\int_0^1 xe^{x^3} + \frac{2x}{2 - x^3}\, dx.$$

3. (10 points) A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ that lies in the first quadrant. Compute the mass of the lamina if the density function is $\rho(x, y) = x + y^2$.

4. (10 points) Calculate the value of the multiple integral

$$\int\int_D \frac{1}{1+x^2}\,\mathrm{d}A,$$

where D is the closed triangular region with vertices $(0,0)$, $(1,1)$, and $(0,1)$.

5. (8 points) Find the linear approximation of the function $f(x, y) = x \sin(5x - 3y)$ at $(3, 5)$ and use it to approximate $f(3.02, 4.9)$.

6. (12 points) Let T be the triangular region in the xy-plane with vertices $(0, 0)$, $(3, 0)$ and $(3, 6)$. Calculate the global maximum value of the function $F(x, y) = 2x^2 - x^2y + y^2$ on the region T.

7. (12 points) Consider the surface S determined by the equation

$$x^2 + 2y^2 - 2z^2 = 1$$

and the path in $\mathbf{R}^3$ given by the vector function

$$\mathbf{r}(t) = \left\langle \sqrt{3}\sin(t), \frac{\sqrt{3}}{\sqrt{2}}\cos(t), 1 \right\rangle.$$

(a) Find the tangent plane to S at the point $(1, 1, 1)$.

(b) Does the path $\mathbf{r}(t)$ lie in the surface S? (As always, you must justify your answer.)

(c) Find $\mathbf{B}(t)$, the binormal vector for $\mathbf{r}(t)$.
(HINT: Do this by thinking about what the curve *looks like* rather than doing a lengthy computation.)

8. (12 points) Luke and Leia are participating in a space race. At $t = 0$ (when the race begins), their ship is at the origin, traveling with speed 1 in the direction of the positive z-axis. Their acceleration at time t is given by $\mathbf{a}(t) = \langle \cos(t), \sin(2t), 8 \rangle$.

   (a) What is the position $\mathbf{r}(t)$ that their ship has at any given time t?

   (b) What is the speed of the space ship when they pass through the plane $z = \pi\left(\pi + \frac{1}{2}\right)$?

9. (12 points) A particle follows the curve $\mathbf{r}(t) = \left\langle \frac{4}{t+1}, t^3, t^4 + 1 \right\rangle$ for time $t \geq 0$.

   (a) What is the curvature at $t = 1$?

   (b) Find the point $P(x, y, z)$ on the curve such that the normal plane to the curve at P is parallel to the plane

$$-\frac{1}{9}x + 3y + 8z = 0.$$