MATH 126 – FINAL EXAM Hints and Answers
SPRING 2011

1. HINT: Let $\vec{n}_1 = \langle -1, 1, 4\rangle$ and $\vec{n}_2 = \langle 1, -3, 2\rangle$, normal vectors for the two planes. A direction vector for the line of intersection is $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 14, 6, 2\rangle$. To find a point on the line, you may assume, for instance, that the line intersects the xy-plane and find the point on both planes with z-coordinate 0: $(-18, -11, 0)$.

   ANSWER: (other answers are possible) $x = -18 + 14t,\ y = -11 + 6t,\ z = 2t$

2. (a) ANSWER: $\kappa(\pi) = \dfrac{2}{1+a^2}$

   (b) ANSWER: $a = 9$

3. (a) ANSWER: $z = y + 1$

   (b) HINT: $\dfrac{\partial z}{\partial x} = 3x^2y^2$ and $\dfrac{\partial z}{\partial y} = 2x^3y$. So, a normal vector of the tangent plane is $\langle 3x_0^2y_0^2, 2x_0^3y_0, -1\rangle$. This must be parallel to $\langle 3, 18, -1\rangle$.
   ANSWER: $\left(3, \frac{1}{3}, 3\right)$ and $\left(-3, -\frac{1}{3}, -3\right)$

4. HINT: Since $xyz = 100$, $z = \dfrac{100}{xy}$. So, $S = x + 2y + \dfrac{300}{xy}$. The only critical point is $(2\sqrt[3]{75}, \sqrt[3]{75})$. You can use the second derivative test to show that this gives a local minimum. Since it is the only critical point, this gives the global minimum.

   ANSWER: The minimum possible value of S is $\dfrac{450}{(75)^{2/3}}$.

5. (a) NOTE: The question as posed is ambiguous. The intended region is bounded *above* by $y = 5 - 4x^2$ and *below* by $y = x^2$.
   HINT: $\displaystyle\iint_R xy\, dA = \int_0^1 \int_{x^2}^{5-4x^2} xy\, dy\, dx.$
   ANSWER: $\dfrac{5}{2}$

   (b) HINT: You must change the order of integration:

$$\int_0^3 \int_{y^2}^9 ye^{x^2}\, dx\, dy = \int_0^9 \int_0^{\sqrt{x}} ye^{x^2}\, dy\, dx.$$

   ANSWER: $\dfrac{e^{81}-1}{4}$

6. HINT: area$= \displaystyle\int_0^{\pi/2} \int_{2\cos\theta}^{1+\cos\theta} r\, dr\, d\theta.$

   ANSWER: $1 - \dfrac{\pi}{8}$

7. (a) ANSWER: $T_3(x) = 1 + 2(x-2) + 2(x-2)^2 + \dfrac{4}{3}(x-2)^3$

   (b) ANSWER: $|f(x) - T_3(x)| \le \dfrac{16e^2}{24}$

8. (a) ANSWER: $\cos(2x^3) = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} 2^{2k} x^{6k}$

(b) ANSWER: 0.7656

(a) ANSWER: $\dfrac{1}{1+4x} - \dfrac{1}{6x-3} = \displaystyle\sum_{k=0}^{\infty} \left[(-1)^k 4^k + \frac{2^k}{3} \right] x^k$

(b) ANSWER: $\dfrac{4}{3} - \dfrac{10}{3}x + \dfrac{52}{3}x^2$

(c) ANSWER: $I = \left(-\dfrac{1}{4}, \dfrac{1}{4} \right)$