This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.

This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.

Give your answers in exact form. Do not give decimal approximations.

In order to receive credit, you must show your work.

Place a box around YOUR FINAL ANSWER to each question.

If you need more room, use the backs of the pages and indicate to the reader that you have done so.

Raise your hand if you have a question.

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1. (10 points) Find parametric equations of the line of intersection of the planes
\[-x + y + 4z = 7 \text{ and } x - 3y + 2z = 15.\]
2. (10 points) Let $a$ be a positive real number.

(a) Find the curvature of the vector function

$$\vec{r}(t) = \langle 2 \cos t, a \sin t, t \rangle$$

at $t = \pi$. Simplify as much as possible.

(b) Find the positive value of $a$ for which $\vec{r}(t)$ has curvature $\frac{1}{41}$ at $t = \pi$. 
3. (12 points)

(a) Find the equation of the tangent plane to the surface 
\[ z = x^2y + e^y - \frac{y}{x} \]
at the point (1, 0, 1).

(b) Find all points \((x_0, y_0, z_0)\) on the surface \(z = x^2y^2\) at which the tangent plane is parallel to the plane 
\[ 3x + 18y - z = 0. \]
4. (10 points) Suppose $x, y,$ and $z$ are positive numbers. What is the minimum possible value of

$$S = x + 2y + 3z$$

if $xyz = 100$? (Include in your solution some verification that your answer is the minimum.)
5. (12 points)

(a) Let $R$ be the region in the first quadrant bounded by $y = 5 - 4x^2$ and $y = x^2$. Evaluate the integral

$$\int \int \limits_{R} xy \, dA$$

(b) Evaluate the integral:

$$\int_{0}^{3} \int_{y^2}^{9} ye^{x^2} \, dx \, dy$$
6. (10 points) Find the area of the region in the first quadrant inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 2 \cos \theta$. 
Let \( f(x) = e^{2x-4} \).

(a) Give the third Taylor polynomial \( T_3(x) \) for \( f(x) \) based at \( b = 2 \).

(b) Use Taylor’s Inequality to bound the error \(|f(x) - T_3(x)|\) on the interval \( I = [1, 3] \).
8. (12 points)

(a) Find the Taylor series for \( \cos(2x^3) \) based at \( b = 0 \). Write your answer in sigma notation.

(b) Use the first three nonzero terms of the Taylor series in part (a), to approximate the value of the integral
\[
\int_0^1 \cos(2x^3) \, dx.
\]
(Give your final answer to four digits after the decimal point).
9. (12 points)

(a) Find the Taylor series for \( f(x) = \frac{1}{1 + 4x} - \frac{1}{6x - 3} \) based at \( b = 0 \). Write your final answer using one sigma sign.

(b) Give the first three non-zero terms of the Taylor series from part (a).

(c) Give an interval \( I \) where the Taylor series from part (a) converges. (Show your work)