Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 6 double-sided pages, including this cover sheet. There is one blank page at the front and two blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	12	
5	14	

Problem	Total Points	Score
6	12	
7	12	
8	14	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. *(12 points)* Let $\mathcal{P}$ be the plane given by the equation

$$-x + y + z = 2.$$

The point $Q(2, 1, -1)$ is not on the plane $\mathcal{P}$. Let L_1 be the line through Q and orthogonal to $\mathcal{P}$, and let R be the point of intersection of L_1 and $\mathcal{P}$. Let W be another point $(-1, 3, 1)$.

Find parametric equations of the line L_2 that passes through points W and R.

Parametric equations: ______________________________

2. *(6 points per part)* **Parts (a) and (b) of this question are unrelated.**

   (a) Find the equation for the quadric surface consisting of all points that are equidistant to the plane $y = 3$ and the point $(0, 1, 2)$. Then identify the quadric surface.

   Equation for surface: ______________________

   Name of surface: ______________________

   (b) Let P be the point where $\mathbf{r}(t) = \left\langle t^2 + t, t^3, \frac{-3}{2}t^2 - 2t \right\rangle$ intersects the plane $y = -8$.

   Find the curvature of $\mathbf{r}(t)$ at P.

   Curvature: ______________

3. *(12 points)* Find the equation of the plane tangent to the surface

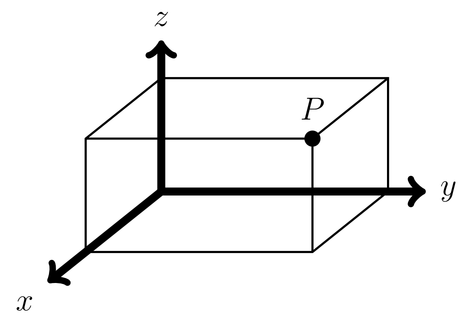
$$z = x^2y - \sqrt{3x - y} - \frac{x}{y}$$

at the point $(6, 2, 65)$. **Write your answer in the form $Ax + By + Cz = D$.**

Tangent plane equation: ______________________________

4. *(12 points)* Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and the vertex opposite the origin (i.e. the one marked P in the picture below) on the surface $z = 20 - x^2 - y$.

   In order to receive full credit, you must show that your answer really is the maximum.



Maximum volume: ______________________

5. *(7 points per part)* **Part (a) and (b) of this question are not related.**

(a) Compute $\displaystyle\int_1^5 \int_{\sqrt{y-1}}^2 \frac{x^2}{1+x^5}\,dx\,dy$ (Hint: reverse the order of integration)

Answer = ______________

(b) Set up (but **do not evaluate**) an integral for the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $y + z = 4$ and $z = 0$.

$$\text{Volume} = \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \, d\square \, d\square$$

6. *(6 points per part)* A lamina D occupies the region **in the first quadrant** that lies inside the circle $x^2 + y^2 = 4$ and outside the circle $x^2 + (y-2)^2 = 4$.

   The density at any point is $\rho(x, y) = \dfrac{x}{\sqrt{x^2 + y^2}}$.

   (a) Find the mass of D.

   (Hint: Set up and evaluate a double integral in polar coordinates.)

   Mass = ____________________

   (b) Find $\overline{y}$, which is the y-coordinate of the center of mass.

   $\overline{y}$ = ____________________

7. *(6 points per part)* Let $f(x) = \sin(x-1) + x^2 + e^x$.

   (a) Find the second Taylor polynomial $T_2(x)$ for f based at $b = 1$.

   $T_2(x) =$ ____________________

   (b) Find an upper bound for $|f(x) - T_2(x)|$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$.

   In order to receive credit, you must justify your answer.

   Upper bound = ____________________

8. For this problem, you may use the following basic Taylor series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(a) *(6 points)* Use the list of Basic Series to find the Taylor series for $f(x) = x^2 \arctan(-3x)$ based at $b = 0$. (Use Σ notation.)

Taylor series: ______________________________

(b) *(3 points)* Find the open interval on which the series in part (a) converges.

Interval of convergence: ____________________

(c) *(5 points)* Find $f^{2023}(0)$. (That is, the 2023th derivative of f at 0.) Give an exact answer.

$f^{(2023)}(0) =$ ____________________

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