Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 9 problems on 7 double-sided pages, including this cover sheet. There is one blank page at the front and three blank pages at the back reserved for scratch work or extra space.
- This exam is closed book. You may use one $8\frac{1}{2}$" × 11" sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the back of the first page or either side of the last page **and indicate that you have done so**. If you *still* need more room, ask for more scratch paper.
- **Do not write within 1 centimeter of the edge of the page.**
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	9	
3	11	
4	12	
5	10	

Problem	Total Points	Score
6	8	
7	14	
8	14	
9	10	
Total	100	

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle “see first page” below a problem.

1. (*4 points per part*) For parts (a)–(c), let $\mathcal{P}$ be the plane $8x - 4y + z = 3$.

   (a) Find the acute angle between $\mathcal{P}$ and the plane $x = 4$.

   (b) Write parametric equations for the line of intersection between $\mathcal{P}$ and the xz-plane.

   (c) Find the point on the plane $\mathcal{P}$ that is closest to $(26, -7, 10)$.

2. *(9 points)* The acceleration of a particle at time t is given by the vector function

$$\mathbf{a}(t) = \langle 2t, 4, 45\sqrt{t} \rangle \text{ m/s}^2.$$

The particle is in the same position at time $t = 1$ as it is at time $t = 4$.

What is the **speed** of the particle at time $t = 0$?

3. *(11 points)* Let $f(x, y) = 2x^2 - 2xy + y^2 - 3y$.

   (a) Find and classify the critical points of f.

   (b) Find the global minimum and global maximum of f on the square $0 \leq x \leq 4$, $0 \leq y \leq 4$.

   (c) You do not need to explain your work on this part.

   Does f have a global minimum on $\mathbf{R}^2$? Yes No

   Does f have a global maximum on $\mathbf{R}^2$? Yes No

4. *(6 points per part)* Evaluate the following double integrals:

(a) $\displaystyle\int_0^1 \int_0^{\arctan(x)} \frac{1}{1-\tan(y)}\, dy\, dx$

(b) $\displaystyle\int\int_D \cos(\sqrt{x^2+y^2}) dA,$
where D is the region in the xy-plane given by $x^2+y^2 \leq 4$, $x \geq 0$, and $y \geq 0$.

5. *(10 points)* In this problem, a denotes a positive constant. A flat plate occupies the part of the disk $x^2 + y^2 \leq a^2$ that lies in the first quadrant. The plate has density $\rho(x, y) = xy^2$. The y-coordinate of the center of mass of the plate is $\bar{y} = 1$.

   Find a.

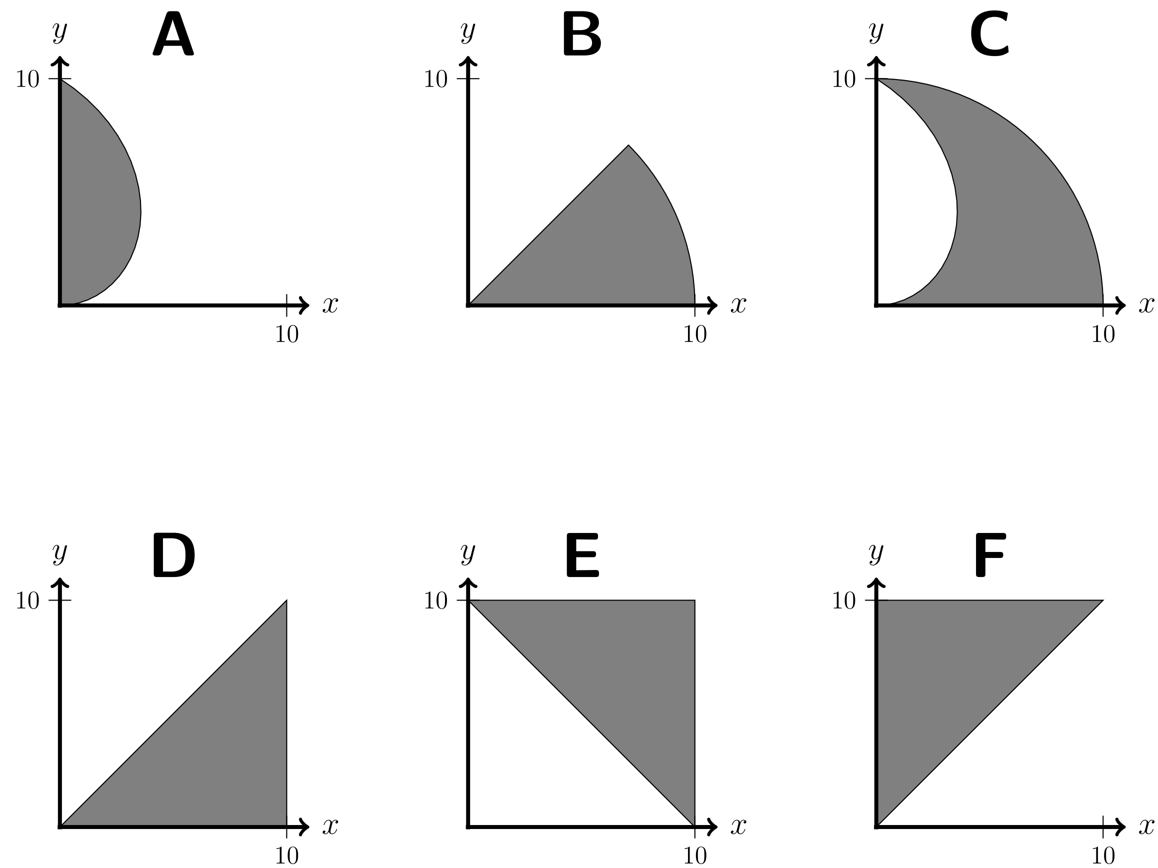
6. *(2 points per part)* Match each of the following area integrals with one of the regions (A)-(F) below, corresponding to its domain of integration. You do not need to justify. (Do NOT compute the integrals)

   (a) $\displaystyle\int_0^{10}\int_0^{\pi/4} r\,d\theta\,dr$ matches region: ______

   (b) $\displaystyle\int_0^{10}\int_0^{y} dx\,dy$ matches region: ______

   (c) $\displaystyle\int_0^{\pi/2}\int_{\left(\frac{20}{\pi}\right)\theta}^{10} r\,dr\,d\theta$ matches region: ______

   (d) $\displaystyle\int_0^{10}\int_x^{10} dy\,dx$ matches region: ______



7. (*7 points per part*) For both part (a) and (b), let $f(x) = \sqrt{(x-1)^5} - \cos(x-2)$.

   (a) Find the second Taylor polynomial, $T_2(x)$, for $f(x)$ based at $b = 2$.

   (b) Use Taylor's inequality to find an upper bound (as sharp as possible) for $|f(x) - T_2(x)|$ on the interval $[1.8, 2.2]$.

8. *(14 points)* For this problem, you may use the following basic Taylor series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

(a) Find the Taylor series for $f(x) = \dfrac{x}{5+x^2}$ based at $b = 0$. Express your answer using $\sum$–notation.

(b) Find the open interval of convergence for the series you found in (a).

(c) Use the Taylor series you found in (a) to find the Taylor series for $g(x) = \ln(5 + x^2)$ based at $b = 0$.

9. *(10 points)* Let $f(x, y) = 100 - x^2 - y^2 + xy$.

   (a) Find a normal vector to the tangent plane to the surface $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$. (Your answer will depend on x_0 and y_0.)

   (b) Find a point P on the portion of the graph of f above the region $-2 \leq x \leq 2$, $-2 \leq y \leq 2$ where the tangent plane at P intersects the xy-plane at the largest possible (acute) angle.

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