• CHECK that your exam contains 8 problems on 8 pages.

• This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.

• Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)

• In order to receive full credit, you must show all of your work.

• Place a box around \textbf{YOUR FINAL ANSWER} to each question.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.

• Raise your hand if you have a question.
1. (12 points) Below are a series of multiple choice questions. Circle the correct answer. You do not need to show your work.

(a) Let \( \mathcal{P} \) be the plane given by the equation \( 2x + y - z = 5 \). Choose the sentence that best describes the plane \( \mathcal{Q} \) given by the equation \( 2x + y - z = 10 \).

   a. \( \mathcal{Q} \) is parallel to \( \mathcal{P} \).  
   b. \( \mathcal{Q} \) is equal to \( \mathcal{P} \).  
   c. \( \mathcal{Q} \) intersects \( \mathcal{P} \) in a line.

(b) Consider the set of points \( C \) whose distance to the origin is twice the distance to the \( x \)-axis. Which of the following best describes \( C \)?

   a. a cone  
   b. an elliptic paraboloid  
   c. a hyperboloid of two sheets  
   d. none of these

(c) Suppose \( |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| > 0 \). Then \( \mathbf{a} \) and \( \mathbf{b} \) are

   a. parallel.  
   b. perpendicular.  
   c. both the zero vector.

(d) Which of the following surfaces does the curve \( \mathbf{r}(t) = \langle t, t, 2t^2 \rangle \) lie on?

   a. \( x^2 + y^2 = z^2 \)  
   b. \( x = 0 \)  
   c. \( x^2 + y^2 = z \)  
   d. all of these  
   e. none of these
2. (12 points) Consider the vector function \( r(t) = (t, \cos(t), \sin(t)) \).

(a) Compute the velocity vector \( v(t) \) and the unit tangent vector \( T(t) \).

(b) Compute the unit normal vector \( N(t) \).

(c) Compute the curvature of \( r(t) \) at the point \( t = 1209.17 \).
3. (12 points) Find and classify all critical points of

\[ f(x, y) = 2x^3 + 9xy^2 + 15x^2 + 27y^2. \]
4. (11 points) Suppose you need to know an equation of the plane tangent to a surface $S$ at the point $P(-7, 4, -33)$. You don’t have an equation for $S$ but you know that the following curves lie on $S$:

$$r_1(t) = \langle 2t - 9, 5 - t, -3t^2 + 26t - 56 \rangle \text{ and } r_2(u) = \langle -\sqrt{u}, \sqrt{u - 33}, -33 \rangle.$$ 

Find the equation of the plane tangent to $S$ at $P$.
Simplify your final answer to the form $z = Ax + By + C$. 


5. (12 points) Evaluate the definite integral

\[ \int_0^1 \int_{e^y}^e \frac{\sqrt{1 + (\ln x)^2}}{x} \, dx \, dy. \]
6. (14 points) Find the volume of the solid that is above the $xy$-plane, within the sphere $x^2 + y^2 + z^2 = 1$, and below the cone $z = \sqrt{x^2 + y^2}$. 
7. (15 points) Let \( f(x) = \sqrt{x^3} \).

(a) Find the second Taylor polynomial \( T_2(x) \) based at \( b = 1 \).

(b) Find an upper bound for \( |T_2(x) - f(x)| \) on the interval \([1 - a, 1 + a]\).
   Assume \( 0 < a < 1 \). Your answer should be in terms of \( a \).

(c) Find a value of \( a \) such that \( 0 < a < 1 \) and \( |T_2(x) - f(x)| \leq 0.004 \) for all \( x \) in \([1 - a, 1 + a]\).
8. (12 points) The Basic Taylor Series are:

\[
\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k + 1)!} \quad \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k.
\]

(a) Use the list of Basic Taylor Series to find the Taylor series for \(f(x) = x^4 \arctan(x^3)\) based at \(b = 0\). (Use \(\Sigma\)-notation.)

(b) Find the open interval on which the series in part (a) converges.

(c) Find \(f^{(2017)}(0)\). (That is, the 2017\(^{th}\) derivative of \(f\) at 0.) Give an exact answer.