Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, non-programmable scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	10	
4	10	
5	14	

Problem	Total Points	Score
6	10	
7	8	
8	12	
9	12	
Total	100	

1. (12 points) The acceleration of a particle is given by $\vec{a}(t) = \langle 2(1+t)^{-1/2}, 0, 2t-5 \rangle$, and its initial velocity is $\vec{v}(0) = \langle 4, 1, 0 \rangle$.

   (a) Determine the velocity $\vec{v}(t)$ of the particle at any time t.

   (b) Find the tangential and normal components of the acceleration at $t = 3$.

   (c) Set up (but DO NOT EVALUATE) the integral that gives the distance traveled by the particle from $t = 0$ to $t = 3$.

2. (12 points) Consider two curves with the following parametric equations:

$$\vec{r}_1(t) = \langle t+4, t+2, t^2-23 \rangle \text{ and } \vec{r}_2(s) = \langle s^2, 4s-5, 2\sqrt{s-2} \rangle.$$

(a) These curves intersect at a point P. Find this point.

(b) Find the angle between the tangent lines to the curves at the point P. (Give the angle in degrees, rounding your answer to two digits after the decimal.)

(c) Find the equation of the normal plane to $\vec{r}_1(t)$ at the point P.

3. (10 points) Let ℓ be the line through the point $Q(1, 2, 0)$ and orthogonal to the plane

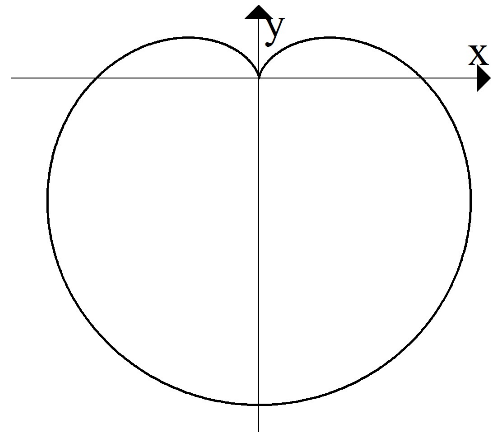
$$x - y + 2z = 10.$$

Let $\mathscr{P}$ be the plane that goes through the points $A(1, 3, 2)$, $B(-1, 3, 0)$, and $C(0, -1, 4)$.

Find the point of intersection of the line ℓ and the plane $\mathscr{P}$ or show that no such point exists.

4. (10 points) Consider the cardiod given by the polar function $r = 2 - 2\sin(\theta)$ (shown below).

   (a) Find the equation for the tangent line at the negative x-intercept.



   (b) Set up (but DO NOT EVALUATE) a double integral in polar coordinates that represents the area inside this cardiod and outside the circle centered at the origin with radius 2.

5. (14 points) Consider the function $f(x, y) = \sin(x^2 + y^2)$. Let D be the unit disk:

$$D = \{(x, y) : x^2 + y^2 \leq 1\}.$$

   (a) Find all critical points of $f(x, y)$ in D and classify each as a local maximum, a local minimum, or a saddle point.

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

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$$f(x, y) = \sin(x^2 + y^2) \text{ and } D = \{(x, y) : x^2 + y^2 \leq 1\}.$$

(b) Find the absolute maximum and minimum values of f on D.

(c) Evaluate the integral $\iint_D f(x, y)\, dA$.

6. (10 points) Evaluate the integral

$$\int_0^1 \int_{e^y}^{e} \sin(x \ln x - x)\, dx\, dy.$$

7. (8 points) Find the equation of the plane tangent to the surface $x^3 + y^4 + z^2 = 0$ at the point $(-1, 0, 1)$.

8. (12 points) Let $f(x) = e^{-2x} - 3x^2$.

   (a) Find the second Taylor polynomial for $f(x)$ based at $b = 0$.

   (b) Give an upper bound for the error $|T_2(x) - f(x)|$ on the interval $[-1, 1]$.

   (c) There is a positive value of x near 0 that satisfies the equation $e^{-2x} = 3x^2$. Use your answer to part (a) to approximate this value of x.

9. (12 points)

   (a) Write out the first four non-zero terms of the Taylor series for $\cos(2x)$ based at $b = 0$.

   (b) Use part (a) and the identity $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ to write out the first four non-zero terms of the Taylor series for $\cos^2(x)$ based at $b = 0$.

   (c) Use your answer to part (b) to compute

$$\lim_{x \to 0} \frac{\cos^2 x - (1 - x^2)}{x^4}.$$