Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, non-programmable, scientific calculator. Do not share notes or calculators.
- Give your answers in exact form. Do not give decimal approximations.
- In order to receive credit, you must show your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	12	
7	10	
8	12	
9	12	
Total	100	

1. (12 points) The acceleration vector of a spaceship is

$$\mathbf{a}(t) = \langle 2t, 0, -\sin(t) \rangle \quad \text{for all } t \geq 0$$

and the specified initial velocity and position are

$$\mathbf{v}(0) = \langle 0, 0, 1 \rangle \quad \text{and} \quad \mathbf{r}(0) = \langle 1, 2, 300 \rangle.$$

(a) Find the velocity function of the spaceship.

(b) Find the tangential component of the acceleration.

(c) Compute the ship's position at $t = \dfrac{\pi}{2}$.

2. (12 points) Suppose a 3-D curve is represented by the vector function

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{1}{3}t^3 - 2t, t^2 - 2 \rangle.$$

(a) Find the curvature at time t.

(b) At what point (x, y, z) does the curve have maximum curvature?

3. (10 points) **True/False**. Answer each question with a T for true or F for false. No justification for your answer is needed.

   (a) ______ If the $\mathbf{T}$ and $\mathbf{N}$ vectors of a vector function $\mathbf{r}(t)$ at $t = 0$ are $\mathbf{j}$ and $\mathbf{k}$ respectively, then the $\mathbf{B}$ vector at $t = 0$ is $\mathbf{i}$.

   (b) ______ There is a function $g(x, y)$ such that $g_x(x, y) = x + \sin(xy)$ and $g_y(x, y) = y + \sin(xy)$.

   (c) ______ For any vectors $\mathbf{a}$ and $\mathbf{b}$, $\text{proj}_{\mathbf{a}}\mathbf{b}$ must be orthogonal to $\mathbf{b}$.

   (d) ______ The level curves of $z = \sqrt{9 - x^2 - y^2}$ are circles.

   (e) ______ If $\mathbf{u} = \langle 1, 2, 3\rangle$ and $\mathbf{v} = \langle 3, 4, 5\rangle$, then $\mathbf{u} \times \mathbf{v} = \langle -2, 1, -2\rangle$.

   (f) ______ Planes $x + y + z = 9$ and $x - 3y + 2z = 4$ are perpendicular.

   (g) ______ If D is the domain given by $x^2 + y^2 \leq 4$, then $\pi\sqrt{3} \leq \iint_D \sqrt{4 - x^2 - y^2}\, dA \leq 2\pi$.

   (h) ______ The curvature of a line is positive.

   (i) ______ The cross product $(\text{proj}_{\mathbf{u}}\mathbf{v}) \times \mathbf{u}$ is zero for any two vectors $\mathbf{u}$ and $\mathbf{v}$.

   (j) ______ The scalar projection $\text{comp}_{\mathbf{u}}\mathbf{v}$ can be positive, or zero, or negative.

4. (10 pts) Find the equation of the plane that contains the point $(0, 0, 0)$ and the line of intersection of the two planes $x - 2y - z = 5$ and $4x + 4y + 14z = 2$.

5. (10 pts) Use the linear approximation of $f(x, y) = (yx^2 - y^4)^{4/3}$ at $(x, y) = (3, 1)$ to approximate the value of $f(2.9, 1.05)$.

6. (12 pts) Find the absolute maximum and minimum values of $f(x, y) = xy^2 - 3x + 1$, on the half disk, $D = \{(x, y) \mid y \geq 0, x^2 + y^2 \leq 36\}$.

7. (10 pts) The region R is outside the circle $x^2 + (y-3)^2 = 9$, inside the circle $x^2 + y^2 = 9$ and in the first quadrant.

   (a) Sketch the circles and shade the region R.

   (b) Compute the area of R.

   (c) Find the volume of the solid above the region R and below the plane $z = x$.

8. (12 pts) Given $f(x) = x\ln(1+5x)$, answer the following.

   (a) Compute the second degree Taylor polynomial $T_2(x)$ based at 0.

   (b) Use $T_2(x)$ you found above to estimate $f(0.02)$.

   (c) Find an upper bound for your error from part (b) using Taylor's Inequality (the error formula).

   (d) Compare your answer in (c) with the difference between the value $f(0.02)$ you get from your calculator and your estimate in (b). Which one is more? Why?

9. (12 pts) For $f(x) = x\cos\left(\frac{1}{2}x^2\right)$, do the following.

   (a) Find the Taylor series for $f(x)$ based at 0. Write your answer in Σ notation. Also, give the first 4 non-zero terms of the series.

   (b) What is the tenth Taylor polynomial $T_{10}(x)$ for this function?

   (c) Compute $f^{(17)}(0)$.