• This exam contains 9 problems on 10 pages. CHECK THAT YOU HAVE A COMPLETE EXAM.

• This exam is closed book. You may use one \(8\frac{1}{2} \times 11\) sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.

• Give your answers in exact form. Do not give decimal approximations.

• In order to receive credit, you must show your work.

• Place a box around [YOUR FINAL ANSWER] to each question.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so.

• Raise your hand if you have a question.

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1. [12 points total] Let $f(x) = e^{\cos(x)}$.

(a) [6 points] Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 0$.

(b) [6 points] Give a bound on the error $|f(x) - T_2(x)|$ for $x$ in the interval $-0.1 \leq x \leq 0.1$. 
2. [10 points total] Let \( f(x) = \ln(2 + 3x^4) \).

Find the coefficient on \( x^{44} \) in the Taylor series for \( f(x) \) based at \( b = 0 \).
3. [15 points total]

(a) [5 points] Evaluate the double integral \( \int_0^1 \int_y^{y^2+1} xy^2 \, dx \, dy \).

(b) [5 points] Evaluate the double integral \( \int_0^2 \int_{x^2}^{4} x \sin(y^2) \, dy \, dx \).
(c) [5 points] Let $R$ be the region which lies in the first quadrant below the line $y = x$ and inside the disk $x^2 + y^2 = 25$. The region $R$ is shown in the figure below. Evaluate the integral

$$
\iint_R xy^2 \, dA.
$$
4. [10 points total] Find the equation for the plane containing the point $P(2, 2, 3)$ and the line given by the parametric equations $x = 4 + t$, $y = 3 + 2t$, $z = 4 + 3t$. 
5. [10 points total] Consider the function \( z = f(x, y) = e^{x+y} + \tan^{-1}(x + y^2) \).

(a) [4 points] Find the two first partial derivatives of \( f(x, y) \).

(b) [4 points] Find the linear approximation to \( f(x, y) \) at the point \((-1, 1)\). Simplify your answer as much as possible.

(c) [2 points] Use the linear approximation you found in part (a) to estimate \( f(-0.5, 1.2) \).

**NOTE:** Do not simply evaluate the function using your calculator!
6. [10 points total] Let \( g(x, y) \) be the distance from the origin to the point \( P(x, y, f(x, y)) \) on the surface \( z = f(x, y) = \sqrt{(x + 2y - 3)^2 + 1} \)

(a) [2 points] Find a formula for \( g(x, y) \).

(b) [8 points] Find the point \( P \) on the surface \( z = f(x, y) \) that is nearest the origin. 
   HINT: instead of minimizing \( g(x, y) \), minimize its square: \( h(x, y) = (g(x, y))^2 \).
7. [10 points total] A particle starts moving from the point \((0, 2, 1)\) with initial velocity \(\vec{v}(0) = \langle -1, 2, 0 \rangle\) and acceleration \(\vec{a}(t) = \langle 2t, 1, 2 - 6t \rangle\).

(a) [4 points] Determine the velocity \(\vec{v}(t)\) and location \(\vec{r}(t)\) of the particle at any time \(t\).

(b) [4 points] Find the tangential and normal components of the acceleration at \(t = 1\).

(c) [2 points] Find the curvature of the particle’s path at \(t = 1\).
8. [11 points total] Consider two curves with the following parametric equations:

\[ \mathbf{r}_1(t) = (\ln(t - 1), \cos(t - 2), t) \quad \text{and} \quad \mathbf{r}_2(u) = (u^2 - 1, e^{u-1}, u + 1). \]

(a) [4 points] Find a value of \( t \) and a value of \( u \) so that \( \mathbf{r}_1(t) = \mathbf{r}_2(u) = (0, 1, 2) \).

(b) [5 points] Find the angle between the tangent lines to the curves at the point \( P(0, 1, 2) \).

(c) [2 points] Set up, but do not evaluate, the integral for the arclength of the curve \( \mathbf{r}_2 \) from \( P(0, 1, 2) \) to \( Q(3, e, 3) \).
9. [12 points total] Consider the function

\[ f(x, y) = \sqrt{x^2 + y^2 - 2y + 2}. \]

(a) [4 points] Sketch the level sets \( f(x, y) = k \) for \( k = 1 \) and \( k = \sqrt{2} \).

(b) [4 points] Write down the equation of the tangent plane to the surface \( z = f(x, y) \) at the point \( P(2, 1, \sqrt{5}) \).

(c) [4 points] The surface \( z = f(x, y) \) is a quadric surface. Identify it.