Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems on 10 pages. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Give your answers in exact form. Do not give decimal approximations.
- In order to receive credit, you must show your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 10 | |
| 5 | 10 | |

| Problem | Total Points | Score |
|---|---|---|
| 6 | 10 | |
| 7 | 10 | |
| 8 | 11 | |
| 9 | 12 | |
| Total | 100 | |

**1.** [**12 points total**] Let $f(x) = e^{\cos(x)}$.

(a) [**6 points**] Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 0$.

(b) [**6 points**] Give a bound on the error $|f(x) - T_2(x)|$ for $x$ in the interval $-0.1 \leq x \leq 0.1$.

**2.** [**10 points total**] Let $f(x) = \ln(2 + 3x^4)$.

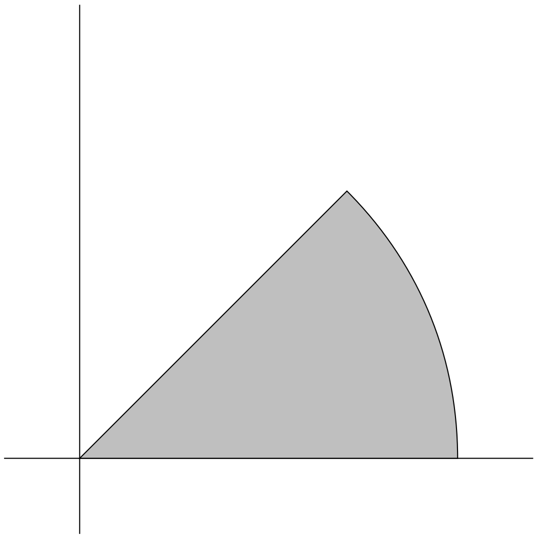
Find the coefficient on $x^{44}$ in the Taylor series for $f(x)$ based at $b = 0$.

**3.** [**15 points total**]

(a) [**5 points**] Evaluate the double integral $\int_0^1 \int_y^{y^2+1} xy^2 \, dx \, dy.$

(b) [**5 points**] Evaluate the double integral $\int_0^2 \int_{x^2}^4 x \sin(y^2) \, dy \, dx.$

(c) [**5 points**] Let $R$ be the region which lies in the first quadrant below the line $y = x$ and inside the disk $x^2 + y^2 = 25$. The region $R$ is shown in the figure below. Evaluate the integral $\displaystyle\iint_R xy^2\,dA$.



**4.** [**10 points total**] Find the equation for the plane containing the point $P(2,2,3)$ and the line given by the parametric equations $x = 4 + t$, $y = 3 + 2t$, $z = 4 + 3t$.

**5.** [**10 points total**] Consider the function $z = f(x, y) = e^{(x+y)} + \tan^{-1}(x + y^2)$.

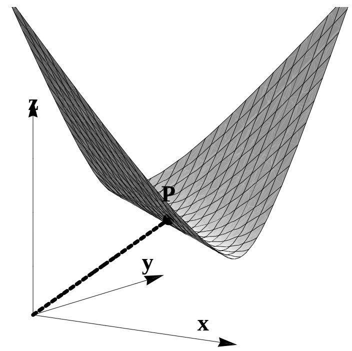
(a) [**4 points**] Find the two first partial derivatives of $f(x, y)$.

(b) [**4 points**] Find the linear approximation to $f(x, y)$ at the point $(-1, 1)$. Simplify your answer as much as possible.

(c) [**2 points**] Use the linear approximation you found in part (a) to estimate $f(-0.5, 1.2)$.

**NOTE:** *Do not simply evalutate the function using your calculator!*

**6.** [**10 points total**] Let $g(x, y)$ be the distance from the origin to the point $P(x, y, f(x, y))$ on the surface $z = f(x, y) = \sqrt{(x + 2y - 3)^2 + 1}$




(a) [**2 points**] Find a formula for $g(x, y)$.

(b) [**8 points**] Find the point $P$ on the surface $z = f(x, y)$ that is nearest the origin.
HINT: instead of minimizing $g(x, y)$, minimize its square: $h(x, y) = (g(x, y))^2$.

**7.** [**10 points total**] A particle starts moving from the point $(0, 2, 1)$ with initial velocity $\vec{v}(0) = \langle -1, 2, 0 \rangle$ and acceleration $\vec{a}(t) = \langle 2t, 1, 2 - 6t \rangle$.

(a) [**4 points**] Determine the velocity $\vec{v}(t)$ and location $\vec{r}(t)$ of the particle at any time $t$.

(b) [**4 points**] Find the tangential and normal components of the acceleration at $t = 1$.

(c) [**2 points**] Find the curvature of the particle's path at $t = 1$.

**8.** [**11 points total**] Consider two curves with the following parametric equations:

$$\vec{r_1}(t) = \langle \ln(t-1), \cos(t-2), t \rangle \text{ and } \vec{r_2}(u) = \langle u^2 - 1, e^{u-1}, u+1 \rangle.$$

(a) [**4 points**] Find a value of $t$ and a value of $u$ so that $\vec{r_1}(t) = \vec{r_2}(u) = \langle 0, 1, 2 \rangle$.

(b) [**5 points**] Find the angle between the tangent lines to the curves at the point $P(0, 1, 2)$.

(c) [**2 points**] Set up, but do not evaluate, the integral for the arclength of the curve $\vec{r_2}$ from $P(0, 1, 2)$ to $Q(3, e, 3)$.

**9.** [**12 points total**] Consider the function

$$f(x, y) = \sqrt{x^2 + y^2 - 2y + 2}.$$

(a) [**4 points**] Sketch the level sets $f(x, y) = k$ for $k = 1$ and $k = \sqrt{2}$.

(b) [**4 points**] Write down the equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(2, 1, \sqrt{5})$.

(c) [**4 points**] The surface $z = f(x, y)$ is a quadric surface. Identify it.