Math 126 Spring 2010 Final Exam Answers

1. (a) Start with the Taylor series for $\dfrac{1}{1-x}$ and substitute $-\frac{1}{2}x^2$ for x; substitute $3x^2$ for x in the Taylor series for e^x. The resulting Taylor series for $f(x)$ is

$$f(x) = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{2^k} - \frac{3^k}{k!} \right) x^{2k}$$

(b) $|x| < \sqrt{2}$

(c) $T_4(x) = -\frac{7}{2}x^2 - \frac{17}{4}x^4$

2. (a) $\sqrt{386}$ (b) Set $\vec{r}\,'(t) \cdot \vec{r}\,''(t) = 0$ and solve for t. Three solutions: $t = 0$, $t = \pm\frac{\sqrt{7}}{3}$.

3. (a) $T_2(x) = x - 1 + \frac{1}{2}(x-1)^2$

(b) Any interval contained in $(0.69333, 1.30667)$ has the desired accuracy.

4. (a) $-8x + 7y + 10z = 36$

(b) $x = \frac{1}{3} + t, y = -3t, z = \frac{2}{3} + 2t$ is one parametrization of the line.

5. (a) $y = -\dfrac{3\sin 6}{2\cos 4}(x - \sin 4) + \cos 6$

(b)

$$\kappa = \frac{|18\cos 4\cos 6 + 12\sin 4\sin 6|}{(4\cos^2 4 + 9\sin^2 6)^{3/2}}$$

6. $\displaystyle\int_0^1 \int_{-2}^3 (12 - 3x - 2y)\,dy\,dx = \frac{95}{2}$

7. (a) The contours are vertically-shifted copies of $y = x^2$.

(b) $z = -2(x-1) + (y-1)$

8. (a) $(0,0)$ is the only critical point; it is a local minimum.

(b) Parametrize the boundary, e.g. $x = 4\sin t, y = 4\cos t$. The critical values of t are then $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$. Evaluating f at these points, and at $(0,0)$, one finds the absolute maximum to be 24.

9. (a) Calculate $\vec{r} \cdot \vec{r}\,'$; simplifiy it to conclude that it is zero.

(b) $|\vec{r}|^2 = \sin^4 t + \sin^2 t\cos^2 t + \cos^2 t = \sin^2 t(\sin^2 t + \cos^2 t) + \cos^2 t = \sin^2 t + \cos^2 t = 1.$