Math 126 **Final Examination** **Spring 2007**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work on the exam paper, with some explanation in English, if appropriate. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the **back of the previous page** and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Consider the function $f(x) = x^3 + x$.

(a) [**5 points**] Find the second Taylor polynomial T_2 of f based at $b = 1$.

(b) [**5 points**] Use Taylor's inequality to find an interval J around b such that the error $|T_2(x) - f(x)|$ is less than 0.001 for all x in J.

2. (a) [**10 points**] Find the first four terms of the Taylor series based at $b = 0$ for the function

$$f(x) = \frac{e^{x^2} - 1}{x} + \frac{3}{(1-x)^2}$$

3. True/False. Answer each question with a T for true or F for false. In all of the questions below, $\vec{v}$ and $\vec{w}$ represent vectors, $\times$ is the cross-product and $\cdot$ the dot product. No justification for your answer is needed.

(a) [**1 point**] ____ $\vec{v} \times \vec{v} = \vec{0}$ only if $\vec{v} = \vec{0}$.

(b) [**1 point**] ____ $\vec{v} \cdot \vec{v} = 0$ only if $\vec{v} = \vec{0}$.

(c) [**1 point**] ____ $\vec{u} \times (\vec{v} \cdot \vec{w})$ makes sense.

(d) [**1 point**] ____ $\vec{u} \cdot (\vec{v} \times \vec{w})$ makes sense.

(e) [**1 point**] ____ $\vec{v} \cdot \vec{w}$ is a vector that is perpendicular to both v and w.

(f) [**1 point**] ____ $\vec{v} \times \vec{w}$ is a vector that is perpendicular to both v and w.

(g) [**2 points**] ____ For all vectors $\vec{v}$ and $\vec{w}$ we have $(\vec{v} - \vec{w}) \cdot (\vec{v} + \vec{w}) = \vec{v} \cdot \vec{v} - \vec{w} \cdot \vec{w}$

(h) [**2 points**] ____ For all vectors $\vec{v}$ and $\vec{w}$ we have $(\vec{v} - \vec{w}) \times (\vec{v} + \vec{w}) = \vec{v} \times \vec{v} - \vec{w} \times \vec{w}$

4. Consider two planes: $P_1 : 2x - y + z = 1$, $P_2 : x - y + z = 2$.

(a) [**5 points**] Find parametric equations for the line of intersection of the two planes.

(b) [**5 points**] Find an equation of the plane that contains the line of intersection from (a) and the point $(1, 1, -1)$.

5. [**10 points**] What is the maximal curvature of the curve $y = \ln \cos x$?

6. An object is moving so that its position at time t (where $t > 0$) is given by the vector function

$$\overrightarrow{r}(t) = \langle t, \frac{1}{t}, t^2 \rangle.$$

(a) [**7 points**] Find all values of t at which the object's acceleration vector is orthogonal to its velocity vector.

(b) [**3 points**] Find the tangential component of acceleration at these values of t.

7. Consider the parametric curve

$$x = t^4 + t^3$$
$$y = t^2 + t.$$

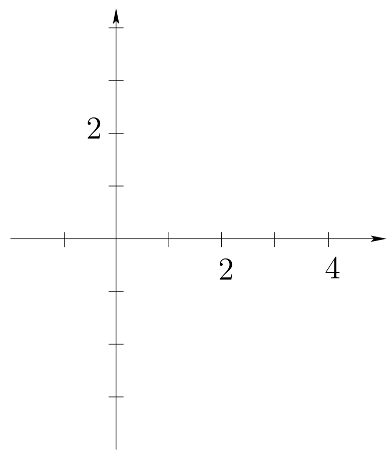
(a) [**5 points**] Find an equation for the tangent line to the curve at the point $(2, 2)$.

(b) [**5 points**] Find all t where the curve has vertical or horizontal tangents.

8. Let

$$f(x, y) = \frac{2x^2 + y^2}{\ln(2x - y)}.$$

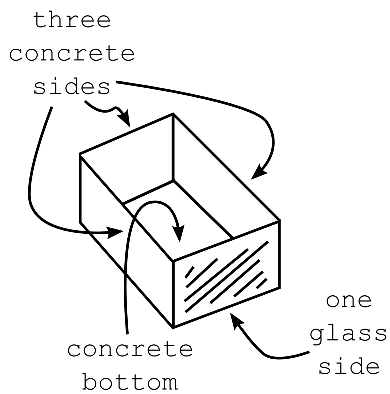
(a) [**3 points**] Find and sketch the domain of f.



(b) [**4 points**] Consider the surface $z = f(x, y)$. Find the equation of the tangent plane to the surface at a point (x_0, y_0, z_0) with $x_0 = y_0 = e$.

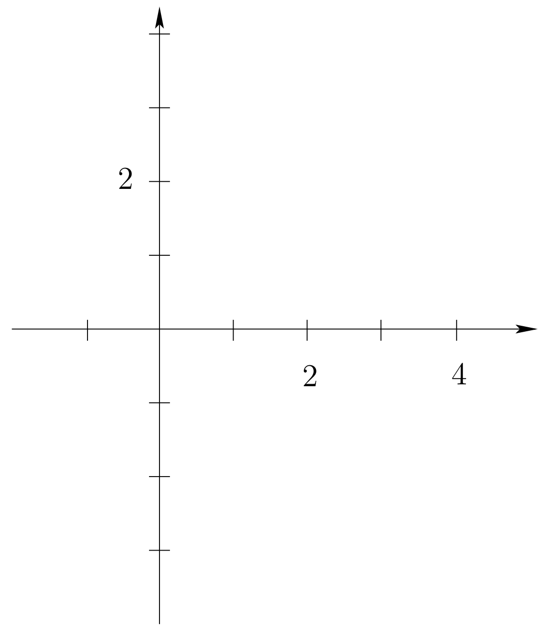
(c) [**3 points**] Using the linear approximation at (e, e) estimate $f(3, 3)$.

9. [**10 points**] You wish to build a large swimming pool in the shaped of a parallelpiped. It will essentially be an open-top box made of concrete. One side, however, will be made of glass, so that the pool can be observed from below ground.



Concrete costs \$15 per square meter, and glass costs \$100 per square meter. If the volume of the pool must be 1000 cubic meters, what should the dimensions be to minimize the cost of the pool?

10. (a) [**4 points**] Draw a picture of the region R between the curves $r = 2\cos\theta$ and $r = 2(1+\cos\theta)$.



(b) [**6 points**] Evaluate the area of R:

$$A(R) = \iint_R 1 dA.$$