Math 126 Final Examination Spring 2006

Your Name

Your Signature

Student ID #

Quiz Section

Professor’s Name

TA’s Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).

- Graphing calculators are not allowed. Do not share notes.

- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.

- **Place a box around YOUR FINAL ANSWER to each question.**

- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

- Raise your hand if you have a question.

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Total 100
1. Let $f(x) = e^{2x-3}$.

(a) [5 points] Find the third-degree Taylor polynomial $T_3(x)$ for $f(x)$ based at $b = 0$.

(b) [5 points] Give an upper bound on the difference between $T_3(x)$ and $f(x)$ on the interval $(-0.5, 0.5)$. 
2.

(a) [6 points] Find Taylor series for the function $f(x) = \ln(2 + 5x)$ centered at $b = 0$.

(b) [4 points] Give an interval on which your series converges and justify your answer.
3. [10 points] Consider a cube such as the one shown in the figure. Consider the line segment connecting corners A and D, and the line segment connecting corners B and C. These are diagonals of the cube.

Find the acute angle between these two diagonals.
4. [10 points] Find an equation of the plane that contains the following two intersecting lines:

\[ x = 4t, \quad y = -1 + 2t, \quad z = 5 - 4t \]

and

\[ x - 1 = 1 - y = \frac{z}{3}. \]
5. **[10 points]** Find the point or points on the curve 

\[ y = x^4 \]

at which the curvature is maximum.
6. Consider the curve given by the vector function

\[ \vec{r}(t) = \langle \sin(\pi t), \cos(\pi t), t^2 - t \rangle. \]

(a) [5 points] Find an equation for the normal plane to the curve when \( t = 2 \). (The normal plane is perpendicular to the tangent vector.)

(b) [5 points] The curve intersects the \( xy \)-plane at two points. Write the formula (but do not evaluate it) for the length of the curve between these two points.
7. A particle is moving so that its position at time \( t > 0 \) is given by the vector

\[
\vec{r}(t) = \langle t^2 + 2, \frac{1}{t}, t \rangle.
\]

(a) [5 points] Find all times when the particle’s velocity and acceleration vectors are orthogonal.

(b) [5 points] Find the magnitude (absolute value) of the tangential and normal components of the particle’s acceleration vector at the times you found in part (a).
8. [10 points] The $x$-coordinate of a particle in the $(x, y)$ plane is calculated based on the particle’s polar coordinates. If the radius $r$ is 2 cm with a possible error of 0.05 cm, and the angle $\theta$ is $\frac{\pi}{6}$ with a possible error of 0.1, use differentials to approximate the range of possible values of the $x$-coordinate.
9. [10 points] Find nonnegative numbers $x$, $y$ and $z$ that minimize the quantity

$$M = x^2 + y^2 + z^2$$

subject to the condition

$$xy^2z^4 = 1.$$
10.

(a) [5 points] Write (but do not evaluate) an iterated integral representing the volume contained inside the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and below the plane $x - z = 0$.

(b) [5 points] Evaluate the iterated integral

\[
\int_{0}^{\frac{x}{2}} \int_{y}^{\frac{x}{2}} \frac{\sin x}{x} \, dx \, dy.
\]