Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive full credit, you must show all of your work on the exam paper (**even** if you could do the work in your head!). Remember to read each problem carefully and answer the questions being asked.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the **back of the previous page** and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	8	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Consider the function $f(x) = (3 + x)^{\frac{1}{2}}$.

(a) [**6 points**] Find the second Taylor polynomial T_2 of f based at $b = 1$.

(b) [**3 points**] Use the Taylor polynomial you computed above to approximate $\sqrt{3.7}$.

(c) [**3 points**] Use Taylor's inequality to find an upper bound for the error in your approximation above.

2. Consider the function $f(x) = \ln(1 + 3x^2)$.

(a) [**6 points**] Find the Taylor series for the function $f(x) = \ln(1 + 3x^2)$ about $b = 0$. Write your answer in summation notation (hint: no differentiation is necessary).

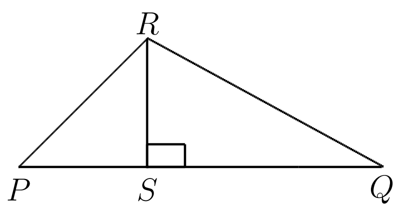
(b) [**2 points**] Find an interval on which the series you just wrote down converges.

3. [**10 points**] Find the plane containing the lines defined parametrically by

$$\vec{\mathbf{r}}_1(t) = (2 + 3t, 1 - t, 5 + 2t) \quad \text{and} \quad \vec{\mathbf{r}}_2(t) = (5 - t, 0 + 2t, 7 - 3t).$$

Give your answer in the form $Ax + By + Cz = D$.

4. Let $P = (1, 2, 0)$, $Q = (-1, 1, 2)$ and $R = (0, 0, 1)$. Let S be the point where the perpendicular line to the side PQ through the point R intersects the line PQ (as indicated, roughly, in the picture below). Answer the following questions about the triangle PQR. (You may answer the following questions in the order (a), (b), (c) or (c), (b), (a).)



(a) [**4 points**] Find the coordinates of the point S.

(b) [**3 points**] Find the height given by the line segment RS.

(c) [**3 points**] Find the area of the triangle PQR.

5. The position of a particle is described parametrically by the function

$$\vec{\mathbf{x}}(t) = (\sqrt{2}\cos t, \sin t + 1, \sin t - 1).$$

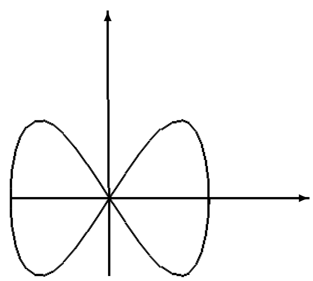
(a) [**3 points**] Compute the magnitude of $\vec{\mathbf{x}}'(t)$ at any time t.

(b) [**2 points**] Compute the arc length of the curve traced out by the particle as t ranges from 0 to 2π.

(c) [**5 points**] Compute the curvature of this curve at any time t.

6. Consider the curve defined parameterically by $x(t) = \cos t$ and $y(t) = \sin t \cos t$.

(a) [**5 points**] Find the equations of both tangent lines at the point where the curve intersects itself (see the picture below).



(b) [**5 points**] Find the area enclosed by the curve.

7. [**10 points**] Captain Kirk and the spaceship USS Enterprise are resting from previous adventures at the point $(1, 2, 0)$ in the xy-plane. At time $t = 0$, they resume their journey (with initial speed zero). Their acceleration is known to be described by $\vec{\mathbf{a}}(t) = (\sqrt{t}, t^2, t - 1)$. At what time and place (specify coordinates) will they return to the xy-plane?

8. Consider the function

$$f(x,y) = \sqrt{4 + 2x^2 - 3y^2}.$$

(a) [**3 points**] Describe and graph the level set of f of level $c = 2$.

(b) [**4 points**] Find an equation of the tangent plane to the surface $z = f(x,y)$ at the point $(2,1,3)$.

(c) [**3 points**] Use the linear approximation to approximate $f(1.9, 1.2)$.

9. (a) [**10 points**] Find the local maximum and minimum values and the saddle points of the function

$$f(x, y) = x^3 - 12x - 6y + y^2 + 1.$$

10. [**10 points**] Consider the region R bounded by a semi-cirle of radius 2, a semi-circle of radius 1, and the x-axis (indicated in the figure below). Compute the average value of the function $f(x,y) = e^{-(x^2+y^2)}$ over the region R.