In this work sheet we'll study the technique of integration by parts. Recall that the basic formula looks like this:

$$\int u\,dv = u \cdot v - \int v\,du$$

1 First a warm-up problem. Consider the integral $\int x\,\sin(3x)\,dx$. Let $u = x$ and let $dv = \sin(3x)\,dx$. Compute du by differentiating and v by integrating, and use the basic formula to compute the original integral. Don't forget the arbitrary constant!

$du = dx \qquad v = -\frac{1}{3}\cos(3x)$

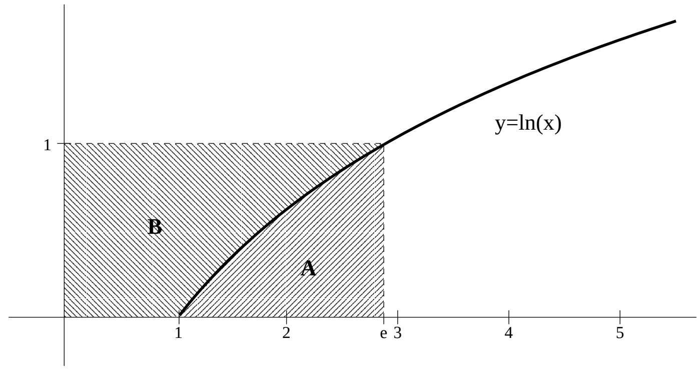
$$\int x\,\sin(3x)\,dx = -\frac{1}{3}x\cos(3x) + \int \frac{1}{3}\cos(3x)\,dx = -\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C$$

2 Compute $\int \ln x\,dx$. (The proper technique is, indeed, integration by parts. What should you take to be u and dv? The choices are pretty limited. Try one and see what happens.)

Let $u = \ln(x)$ *and* $dv = dx$. *Then* $du = \frac{1}{x}\,dx$ *and* $v = x$.

$$\int \ln(x)\,dx = x\ln(x) - \int x \cdot \frac{1}{x}\,dx = x\ln(x) - \int\,dx = x\ln(x) - x + C$$

3 The regions A and B in the figure are revolved around the x-axis to form two solids of revolution.



(a) Before computing the integrals, which solid do you think has a larger volume? Why?

Region B looks larger.

(b) Use the disk method to find the volume of the solid swept out by region A.

$$\int_1^e \pi \ln(x)^2 \, dx$$

$$u = \ln(x)^2 \qquad dv = \pi \, dx$$

$$du = \frac{2\ln(x)}{x} \, dx \qquad v = \pi x$$

$$\int \pi \ln(x)^2 \, dx = \pi x \ln(x)^2 - \int 2\pi \ln(x) \, dx = (*)$$

$$U = \ln(x) \qquad dV = 2\pi \, dx$$

$$dU = \frac{1}{x} \, dx \qquad V = 2\pi x$$

$$(*) = \pi x \ln(x)^2 - 2\pi x \ln(x) + \int 2\pi \, dx = \pi x \ln(x)^2 - 2\pi x \ln(x) + 2\pi x + C$$

The volume is $\left(\pi x \ln(x)^2 - 2\pi x \ln(x) + 2\pi x\right)\Big|_1^e = \pi(e-2) \approx 2.2565$

(c) Use the shell method to find the volume of the solid swept out by region B.

$$\int_0^1 2\pi y e^y \, dy$$

$$u = 2\pi y \qquad dv = e^y \, dy$$

$$du = 2\pi \, dy \qquad v = e^y$$

$$\int 2\pi y e^y \, dy = 2\pi y e^y - \int 2\pi e^y \, dy = 2\pi y e^y - 2\pi e^y + C$$

The volume is $(2\pi y e^y - 2\pi e^y)|_0^1 = 2\pi \approx 6.283$

4 Suppose we try to integrate $1/x$ by parts, taking $u = 1/x$ and $dv = dx$. We have $du = (-1/x^2)\,dx$ and $v = x$, so

$$\begin{aligned}\int \frac{1}{x}\,dx &= \frac{1}{x}\cdot x - \int x \cdot \frac{-1}{x^2}\,dx \\ &= 1 + \int \frac{1}{x}\,dx.\end{aligned}$$

Canceling the integral from both sides, we get the disconcerting result that $0 = 1$. *What went wrong?* What happens if we replace the indefinite integrals by definite integrals, that is, if we try to calculate $\int_a^b \frac{1}{x}\,dx$ by this method?

This issue here is that indefinite integrals are defined only up to an arbitrary constant. The correct equation should look like $0 = 1 + C$

For the second question, $\int_a^b \frac{1}{x}\,dx = 1\big|_a^b + \int_a^b \frac{1}{x}\,dx$. *Here the term* $1\big|_a^b = 1 - 1 = 0$.