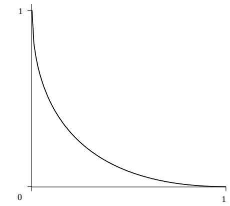
1 If the radius of the big wheel is taken to be one, the part of the astroid in the first quadrant can be shown to have the equation $x^{2/3}+y^{2/3}=1$. Use disks to compute the volume of the solid generated by rotating this portion of the curve around the y-axis.



Compute $\int_0^1 \pi x^2\, dy$ *where* $x^2=\left(1-y^{2/3}\right)^3$.

Thus the integral is $\pi \int_0^1 1-3\,y^{2/3}+3\,y^{4/3}-y^2\,dy=\dfrac{16}{105}\,\pi$.

Hopefully, the students will complain about the algebra here. The method in Problem 2 is less algebraic, but requires a tricky substitution.

2 Use cylindrical shells to compute the volume of the solid generated by rotating the first quadrant portion of the astroid about the x-axis. How does this compare with your answer in Problem 1? Can you explain this geometrically?

Compute $\int_0^1 2\pi\, x\, y\, dy$ *where* $x=\left(1-y^{2/3}\right)^{3/2}$.

The integral is $\int_0^1 2\pi\, y\, \left(1-y^{2/3}\right)^{3/2}\, dy$.

Take $u^3=y^2$ *and* $3u^2\,du=2y\,dy$ *to get* $\int_0^1 3\pi\, u^2\,(1-u)^{3/2}\,du$.

Now take $v=1-u$ *and* $dv=-du$ *to get*

$$-3\pi\int_1^0 (1-v)^2\,v^{3/2}\,dv=3\pi\int_0^1 v^{3/2}-2\,v^{5/2}+v^{7/2}\,dv=\frac{16}{105}\,\pi.$$

The volumes are the same because the astroid curve is symmetric about the line $y=x$.

Notice that shells, here, are more difficult than disks.

3 Use any method you wish to compute the volumes of the solids generated by rotating the first quadrant portion of the astroid about the lines $x=1$ and $y=-1$. **Set up only. Do not compute the integrals.**

I'd use shells for both of these.

For $x=1$ *the integral is* $\int_0^1 2\pi\,(1-x)\,y\,dx=2\pi\int_0^1 (1-x)\left(1-x^{2/3}\right)^{3/2}\,dx=\dfrac{3}{16}\,\pi^2-\dfrac{16}{105}\,\pi$.

For $y=-1$ *the integral is* $\int_0^1 2\pi\,(y+1)\,x\,dy=2\pi\int_0^1 (y+1)\left(1-y^{2/3}\right)^{3/2}\,dy=\dfrac{3}{16}\,\pi^2+\dfrac{16}{105}\,\pi$.

These are both solved in a similar manner to Problem 2. The thing to focus on is the set-up. Make sure the students sketch pictures.