

Name \_\_\_\_\_

Quiz Section \_\_\_\_\_

This worksheet helps you review improper integrals and approximation techniques.

## Improper Integrals

1 Show that  $\int_1^{\infty} \frac{1 - \sin x}{x^2} dx$  converges, using the Comparison Test.

2 Use Problem 1 and the fact that  $\int_1^{\infty} \frac{1}{x^2} dx$  converges to show that  $\int_1^{\infty} \frac{\sin x}{x^2} dx$  converges. Why can't we use the Comparison Test directly on this one?

3 Use Problem 2 and integration-by-parts to show that  $\int_1^{\infty} \frac{\cos x}{x} dx$  converges.

## Approximation Techniques

Let  $f(x) = \frac{\ln(x)}{x}$ . We will use geometric reasoning to see if the Trapezoid Rule gives an overestimate, or an underestimate of the integral  $\int_1^3 f(x) dx$ .

1 Compute  $f'(x)$ . Where is the function increasing and decreasing? (We only care about the interval  $1 \leq x \leq 3$ .)

2 Compute  $f''(x)$ . Is the function concave up or concave down on the interval?

3 Sketch a graph of  $y = f(x)$  on the interval  $1 \leq x \leq 3$ . Sketch the approximate area under the curve given by the Trapezoid Rule with  $n = 4$  subintervals. Is it an overestimate or an underestimate? How is the answer related to Problem 2?