In this work sheet we’ll study the problem of finding the area of a region bounded by curves. We’ll first estimate an area given numerical information. The we’ll use calculus to find the area of a more complicated region.

The Lake

1. The widths, in feet, of a small lake were measured at 40 foot intervals. Estimate the area of the lake.

Area Bounded by Three Curves

2. On the grid below sketch the graphs of \( y = 4, \ y = x^2 \) and \( y = \sqrt{27x} \). (The last one is just a piece of a sideways parabola).

3. Shade the “triangular” region bounded by the graphs of the three functions that lies above the horizontal line.
4 Compute the $x$-coordinate of the left endpoint of the region.

5 Compute the $x$-coordinate of the right endpoint of the region.

6 Note that the top of the region consists of a single curve, but the bottom of the region consists of two different curves. Find the $x$-coordinate where these two curves meet.

7 Sketch in a vertical line at the $x$-coordinate you found in the last problem. This divides the region into two smaller sub-regions.

8 Compute the area of the left sub-region.
Compute the area of the right sub-region. Add the two areas together to get the total area.

Recompute the area using the following trick. Solve for $x$ as a function of $y$ in the two non-constant functions. Find the area by integrating with respect to $y$. Is this easier?