

In this work sheet we'll study the technique of integration by parts. Recall that the basic formula looks like this:

$$\int u \, dv = u \cdot v - \int v \, du$$

1 First a warm-up problem. Consider the integral $\int x \sin(3x) \, dx$. Let $u = x$ and let $dv = \sin(3x) \, dx$. Compute du by differentiating and v by integrating, and use the basic formula to compute the original integral. Don't forget the arbitrary constant!

$$du = dx \quad v = -\frac{1}{3} \cos(3x)$$

$$\int x \sin(3x) \, dx = -\frac{1}{3} x \cos(3x) + \int \frac{1}{3} \cos(3x) \, dx = -\frac{1}{3} x \cos(3x) + \frac{1}{9} \sin(3x) + C$$

2 Compute $\int \ln x \, dx$. (The proper technique is, indeed, integration by parts. What should you take to be u and dv ? The choices are pretty limited. Try one and see what happens.)

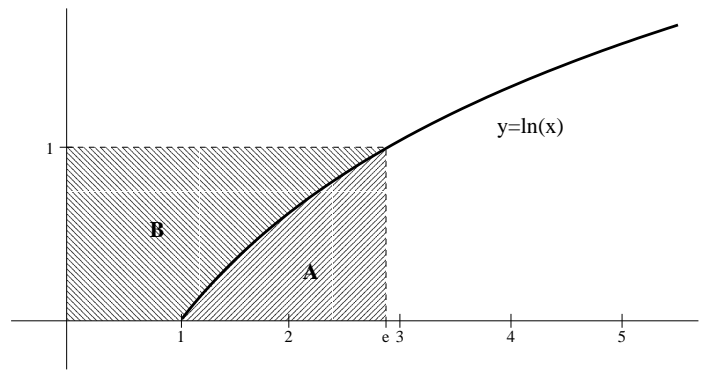
Let $u = \ln(x)$ and $dv = dx$. Then $du = \frac{1}{x} \, dx$ and $v = x$.

$$\int \ln(x) \, dx = x \ln(x) - \int x \cdot \frac{1}{x} \, dx = x \ln(x) - \int dx = x \ln(x) - x + C$$

3 The regions A and B in the figure are revolved around the x -axis to form two solids of revolution.

(a) Before computing the integrals, which solid do you think has a larger volume? Why?

Region B looks larger.



(b) Use the disk method to find the volume of the solid swept out by region A .

$$\int_1^e \pi \ln(x)^2 dx$$

$$u = \ln(x)^2 \quad dv = \pi dx$$

$$du = \frac{2 \ln(x)}{x} dx \quad v = \pi x$$

$$\int \pi \ln(x)^2 dx = \pi x \ln(x)^2 - \int 2\pi \ln(x) dx = (*)$$

$$U = \ln(x) \quad dV = 2\pi dx$$

$$dU = \frac{1}{x} dx \quad V = 2\pi x$$

$$(*) = \pi x \ln(x)^2 - 2\pi x \ln(x) + \int 2\pi dx = \pi x \ln(x)^2 - 2\pi x \ln(x) + 2\pi x + C$$

$$\text{The volume is } \left(\pi x \ln(x)^2 - 2\pi x \ln(x) + 2\pi x \right) \Big|_1^e = \pi(e - 2) \approx 2.2565$$

(c) Use the shell method to find the volume of the solid swept out by region B .

$$\int_0^1 2\pi y e^y dy$$

$$u = 2\pi y \quad dv = e^y dy$$

$$du = 2\pi dy \quad v = e^y$$

$$\int 2\pi y e^y dy = 2\pi y e^y - \int 2\pi e^y dy = 2\pi y e^y - 2\pi e^y + C$$

$$\text{The volume is } (2\pi y e^y - 2\pi e^y) \Big|_0^1 = 2\pi \approx 6.283$$

4 Suppose we try to integrate $1/x$ by parts, taking $u = 1/x$ and $dv = dx$. We have $du = (-1/x^2) dx$ and $v = x$, so

$$\begin{aligned}\int \frac{1}{x} dx &= \frac{1}{x} \cdot x - \int x \cdot \frac{-1}{x^2} dx \\ &= 1 + \int \frac{1}{x} dx.\end{aligned}$$

Canceling the integral from both sides, we get the disconcerting result that $0 = 1$. *What went wrong?* What happens if we replace the indefinite integrals by definite integrals, that is, if we try to calculate $\int_a^b \frac{1}{x} dx$ by this method?

This issue here is that indefinite integrals are defined only up to an arbitrary constant. The correct equation should look like $0 = 1 + C$

For the second question, $\int_a^b \frac{1}{x} dx = 1|_a^b + \int_a^b \frac{1}{x} dx$. Here the term $1|_a^b = 1 - 1 = 0$.