1. (10 points) Evaluate the following indefinite integrals.
(a) (5 points) We start with substitution $u = \pi\sqrt{t}$ so that $du = \frac{\pi}{2\sqrt{t}}dx$. The integral becomes

$$\int \frac{\sin(\pi\sqrt{t})}{\sqrt{t}}\,dt = \frac{2}{\pi}\int \sin(u)\,du = -\frac{2}{\pi}\cos(u) + C = \boxed{-\frac{2}{\pi}\cos(\pi\sqrt{t}) + C}.$$

(b) (5 points) The substitution $x = u^6$ works well since then $\sqrt{x} = u^3$, $\sqrt[3]{x} = u^2$ and $dx = 6u^5\,du$. Thus

$$\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}} = \int \frac{6u^5}{u^3 - u^2}\,du = 6\int \frac{u^3}{u-1}\,du.$$

After doing long division we find that this is equal to

$$6\int \left(u^2 + u + 1 + \frac{1}{u-1}\right)\,du = 6\left[\frac{u^3}{3} + \frac{u^2}{2} + u + \ln|u-1|\right] + C.$$

After substituting back $u = \sqrt[6]{x}$ we get

$$\boxed{2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C}$$

2. **(a)** (5 points) (Note: This is Stewart 7.4 #33. This can also be done by partial fractions.) Let $u = x^4 + 4x^2 + 3$ so that $du = (4x^3 + 8x)\,dx = 4(x^3 + 2x)\,dx$. In changing the limits of integration we note that $x = 0 \implies u = 3$ and $x = 1 \implies u = 8$. Thus

$$\int_0^1 \frac{x^3 + 2x}{x^4 + 4x^2 + 3}\,dx = \frac{1}{4}\int_3^8 \frac{1}{u}\,du = \frac{1}{4}\ln|u|\Big|_3^8 = \boxed{\frac{1}{4}(\ln 8 - \ln 3)} = \boxed{\frac{1}{4}\ln\frac{8}{3}}$$

(b) (5 points) (Note: this can also be done by first integrating by parts.)

$$\int_0^{1/2} \sin^{-1}(x)\,dx$$

We approach this as a trigonometric substitutution using $x = \sin\theta$ so that $dx = \cos\theta\,d\theta$ and $\theta = \sin^{-1}(x)$. Thus

$$\int \sin^{-1}(x)\,dx = \int \theta\cos\theta\,d\theta = \theta\sin\theta - \int \sin\theta\,d\theta = \theta\sin\theta + \cos\theta + C = \sin^{-1}(x)\cdot x + \sqrt{1-x^2} + C$$

where we have used integration by parts to evaluate the θ integral. Therefore

$$\int_0^{1/2} \sin^{-1}(x)\,dx = x\sin^{-1}(x) + \sqrt{1-x^2}\Big|_0^{1/2} = \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}.$$

3. (10 points) Let R be the region that is between the curve $y = \sqrt{x}e^{-x^2}$ and the x-axis, is bounded on the left by the line $x = 1$, and extends infinitely far out to the right. Let S be the solid obtained by rotating R around the x-axis.
Does S have finite volume? If so, find it, and give your answer in exact form. (Note: This is Problem #3 the Winter 2014 Final.)

Using the method of disks we see that the volume is given by the improper integral

$$\int_1^\infty \pi(\sqrt{x}e^{-x^2})^2\,dx = \int_1^\infty \pi x e^{-2x^2}\,dx.$$

We first evaluate the indefinite integral using a substitution $u = -2x^2$ and $du = -4x\,dx$ so that

$$\int \pi(\sqrt{x}e^{-x^2})^2\,dx = -\frac{\pi}{4}\int e^u\,du = -\frac{\pi}{4}e^u + C = -\frac{\pi}{4}e^{-2x^2} + C.$$

Therefore

$$\int_1^\infty \pi(\sqrt{x}e^{-x^2})^2\,dx = \lim_{t\to\infty}\int_1^t \pi(\sqrt{x}e^{-x^2})^2\,dx = \lim_{t\to\infty}\left[-\frac{\pi}{4}e^{-2x^2}\right]\Bigg|_1^t = \lim_{t\to\infty}\left[-\frac{\pi}{4}e^{-2t^2} + \frac{\pi}{4}e^{-2}\right] = \boxed{\frac{\pi}{4}e^{-2}}.$$

4. (10 points) A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

(**Note**: There are many correct ways to do this problem. As mentioned many times in class *weight* is a measurement of force so you do not need to multiply by the acceleration due to gravity.)

The work needed to raise the elevator alone is 1600 lb $\times$ 30 ft $= 48,000$ ft-lb. The work needed to raise the bottom 170 ft of cable is 170 ft $\times$ 10 lb/ft $\times$ 30 ft = 51,000 ft-lb. The work need to raise the top 30 ft of cable is $\int_0^{30} 10x\,dx = 5x^2|_0^{30} = 5\cdot 900 = 4500$ ft-lb. Adding these together, we see that the total work needed is $48,000 + 51,000 + 4,500 = \boxed{103,500 \text{ ft-lb}}$.

5. (10 points) (Note: This is problem 8 in the Autumn 2009 Final).

(a) (4 points) Use Simpson's rule with $n = 4$ subintervals to approximate the integral. Give your answer in *EXACT* form (involving numbers like $\ln(3)$, etc.).
DO NOT GIVE A DECIMAL APPROXIMATION in this part. $\Delta x = \frac{3-1}{4} = \frac{1}{2}$ so we get

$$\int_1^3 \ln x\,dx \approx \boxed{\frac{1}{6}(0 + 4\ln(1.5) + 2\ln(2) + 4\ln(2.5) + \ln(3))}$$

(b) (4 points) Compute the integral exactly.
DO NOT GIVE A DECIMAL APPROXIMATION in this part.

$$\int_1^3 \ln x\,dx = [x\ln x - x]|_1^3 = (3\ln(3) - 3) - (1\ln(1) - 1) = \boxed{3\ln(3) - 2}$$

(c) (2 points) Use your calculator to evaluate your answers in part (a) and part (b) as decimals; round your answers to six decimal digits after the decimal point.

$$\boxed{\text{Simpson} = 1.295322 \qquad \text{Exact} = 1.295837}.$$