

(80 minutes — 100 points)

1. (20 points each) Find the following integrals. For definite integrals please simplify your answer and leave it in exact form. Please show your work clearly.

(a) $\int \frac{\sin x}{\cos^2 x - 5 \cos x + 6} dx.$

(b) $\int_5^\infty \frac{dx}{x^2 \sqrt{x^2 + 25}}.$

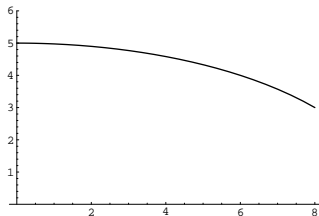
(c) $\int_9^\infty e^{-\sqrt{x}} dx.$

2. (15 points) A well is in the shape of a cylinder of radius 1 meter and depth 8 meters. If it's half full of water, find the work required to remove the water from the well (in other words, to raise it to the level of the top of the well). Please show your work clearly, give your answer in decimal form, and label it with the correct unit. Gravitation acceleration is 9.8 m/sec^2 , and the density of water is 1000 kg/m^3 .

3. (25 points) Use Simpson's rule with $n = 4$ subdivisions to approximate the integral for the length of the arc of the ellipse

$$x^2 + 4y^2 = 100$$

that joins the points $(0, 5)$ and $(8, 3)$. Please show all of your steps in setting up the integral and doing the computations. Give your answer as a decimal.



ANSWERS to Math 125AA,AB,BC midterm, Feb. 22, 2007

1. (a) $\ln\left(\frac{2-\cos x}{3-\cos x}\right) + C.$

(b) $\frac{\sqrt{2}-1}{25}.$

(c) (See 5(b) of thw Week 6 Homework Problems sheet.) $8e^{-3}.$

2. $9800\pi \cdot 24 = 739,000$ joules.

3. $\int_0^8 \sqrt{1 + \frac{x^2}{4(100-x^2)}} dx \approx 8.3611.$