Problem 1 (6+6 points)

a. Find the general anti-derivative $\int \cos^3 \theta \, d\theta$

b. Evaluate the integral $\int_1^2 \frac{x^3}{x^2 + x + \frac{1}{2}} \, dx$
Problem 2 (6+6 points)

a. Find the general anti-derivative \( \int \frac{\sqrt{y - 4}}{y} \, dy \)

b. Does the improper integral \( \int_{0}^{\infty} \frac{1}{x^2 + 4x + 3} \, dx \) converge? If yes, evaluate it.
Problem 3 (4+4 points).

a. Find the average value of the function \( f(x) = \sqrt{x} \) on the interval \( 0 \leq x \leq 2 \).

b. Find the number \( c \) so that the function \( f(x) = \sqrt{x} \) has average value 1 on the interval \([0, c]\).
Problem 4 (8 points).
The following table lists values of two functions, $f_1(x)$ and $f_2(x)$, for different values of $x$. Use the Trapezoid rule to approximate the area between the curves $f_1(x)$ and $f_2(x)$ for $1 \leq x \leq 4$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$</td>
<td>0</td>
<td>.7</td>
<td>1.1</td>
<td>1.3</td>
<td>1.2</td>
<td>.9</td>
<td>.5</td>
</tr>
<tr>
<td>$f_2(x)$</td>
<td>0</td>
<td>-.1</td>
<td>-.2</td>
<td>-.1</td>
<td>0</td>
<td>.1</td>
<td>.3</td>
</tr>
</tbody>
</table>
Problem 5 (10 points).
The line \( y = 3x \), for \( 0 \leq x \leq 1 \), is rotated about the \( y \)-axis to form a cone (units are in feet). The cone is filled with melted ice cream, which weighs 59.2 lb/ft\(^3\). How much work does it take to pump all of the ice cream up to the height \( y = 10 \)?