Math 125 F - Winter 2016
Midterm Exam Number One
January 28, 2016

Name: ____________________________  Student ID no. : ________________

Signature: ____________________________  Section: __________

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- This exam consists of SIX problems on SIX pages, including this cover sheet.
- Show all work for full credit.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you run out of room, write on the back of the page, but indicate that you have done so!
- You may use one hand-written double-sided 8.5” by 11” page of notes.
- You may use a scientific calculator. Calculators with graphing, differentiation, integration, or algebraic capabilities are not allowed.
- You have 80 minutes to complete the exam.
1. [4 points per part] Compute the indefinite integrals.

(a) \[ \int \left( \sqrt[3]{x} - \frac{2}{\sqrt{1-x^2}} \right) \, dx \]

\[ = \frac{7^{8/7}}{8} - 2 \arcsin(x) + C \]

(b) \[ \int (x^{1.7} + e^{3x}) \, dx \]

\[ = \frac{2.7}{2.7} + \frac{1}{3} \int e^{3x} \, dx = \frac{x^{2.7}}{2.7} + \frac{1}{3} \int e^u \, du = \frac{x^{2.7}}{2.7} + \frac{e^{3x}}{3} + C \]

\[ u = 3x \]
\[ du = 3 \, dx \]

(c) \[ \int \frac{\sin^2(\ln(x)) \cos(\ln(x))}{x} \, dx = \int u^3 \, du \]

\[ u = \sin(\ln(x)) \]
\[ du = \cos(\ln(x)) \cdot \frac{1}{x} \, dx \]

\[ \int \frac{\sin^3(\ln(x))}{3} + C \]
2. [12 points] Compute the area of the region bounded by the following three curves:

\begin{align*}
  y &= \frac{54}{x} \\
  y &= 1 \\
  y &= 2\sqrt{x} \\
  y &= \frac{54}{x}
\end{align*}

It is easier to integrate w.r.t. \( y \):

\[
\int_1^6 \left( \frac{54}{y} - \frac{y^3}{4} \right) \, dy
\]

\[
= \left. \left( 54\ln y - \frac{y^4}{12} \right) \right|_1^6
\]

\[
= \left( 54\ln(6) - \frac{216}{12} \right) + \left( 1 - \frac{1}{12} \right)
\]

\[
= 54\ln(6) - \frac{215}{12}
\]
3. **[10 points]** A remote-controlled tomato is moving along the number line. Its velocity after \( t \) seconds is given by the formula
\[
v(t) = 9 - 3^t.
\]
Compute the total distance traveled by the tomato from time \( t = 0 \) to \( t = 4 \).
(You do not need to simplify your answer.)

\[
\begin{align*}
\text{When does it turn around?} & \quad v(t) = 9 - 3^t = 0 \\
& \quad \downarrow \\
\text{Displacement after } t \text{ seconds:} & \quad t = 2 \\
\mathcal{s}(t) &= \int_0^t (9 - 3^x) \, dx \\
& = \left[ 9x - \frac{3^x}{\ln(3)} \right]_0^t \\
& = 9t - \frac{3^t - 1}{\ln(3)} \\
\mathcal{s}(0) &= 0 \\
\mathcal{s}(2) &= 18 - \frac{8}{\ln(3)} \\
\mathcal{s}(4) &= 36 - \frac{80}{\ln(3)} \\
\text{Total distance} &= \left( 18 - \frac{8}{\ln(3)} - 0 \right) + \left( \left( 18 - \frac{8}{\ln(3)} \right) - \left( 36 - \frac{80}{\ln(3)} \right) \right) \\
& = \frac{64}{\ln(3)}
\end{align*}
\]

4. **[5 points]** Write (but do not simplify) a formula for the \( L_{1000} \) approximation of \( \int_0^2 \sin(x) \, dx \).
(Please use \( \Sigma \)-notation. Do not write out a thousand summands.)

\[
\sum_{i=0}^{999} f(x_i \Delta x) \Delta x = \sum_{i=0}^{999} \sin \left( \frac{2i}{1000} \right) \cdot \frac{2}{1000} = \sum_{i=0}^{999} \sin \left( \frac{i}{500} \right) \left( \frac{1}{500} \right)
\]
5. **[12 points]** Let \( R \) be the region in the \( x-y \) plane below \( y = \sec(x) \tan(x) \) and above \( y = -2 \) from \( x = 0 \) to \( x = \frac{\pi}{4} \).

(a) Write an integral to compute the volume of the solid formed by revolving \( R \) around the line \( y = -2 \).

\[
V = \int_0^{\frac{\pi}{4}} \pi \left( \sec^2(x) + \tan^2(x) \right) dx
\]

(b) Evaluate the integral from part (a).

\[
\begin{align*}
V &= \pi \int_0^{\frac{\pi}{4}} \left( \sec^3(x) + \tan^3(x) \right) dx \\
&= \pi \left[ \left. \frac{\tan^3(x)}{3} + 4 \sec(x) + 4x \right|_0^{\frac{\pi}{4}} \right] \\
&= \pi \left( \frac{\tan^3\left(\frac{\pi}{4}\right)}{3} + 4 \sec\left(\frac{\pi}{4}\right) + 4 \cdot \frac{\pi}{4} \right) - \left( 0 + 4 \cdot 0 \right) \\
&= \pi \left( \frac{1^3}{3} + 4 \sqrt{2} + \pi \right) - \left( 0 + 4 \cdot 0 \right) \\
&= \left( \frac{\pi}{3} + 4 \sqrt{2} + \pi \right) - \left( 0 + 4 \cdot 0 \right) \\
&= \pi \left( \frac{\pi}{3} + 4 \sqrt{2} + \pi \right)
\end{align*}
\]
6. Below is the graph of \( f(x) \), the most beautiful function you’ve ever seen.

Use this graph to answer the following questions.

(a) [3 points] Does \( \int_{-4}^{-1} f(x) \, dx \) exist? Explain, briefly.

[3 points] It only has one (removable) discontinuity, so the Riemann sums converge.

(b) [3 points] Evaluate \( \lim_{n \to \infty} \sum_{i=1}^{n} f\left(2 + \frac{5i}{n}\right) \frac{5}{n} \).

\[
\int_{2}^{7} f(x) \, dx \approx 11 + \frac{9\pi}{4}
\]

(c) [3 points] Let \( h(x) = \int_{0}^{2x} f(3t) \, dt \). Compute \( h'(1) \).

\[ h(x) = g(2x) \text{ so } h'(x) = g'(2x) \cdot 2 = 2f(6x) \]

\[ h'(1) = 2f(6) = 5 \]