Problem 1.
This problem involves a car that moves in a straight line on a racing track. Let t denote the time (measured in seconds) after the start of a race. Suppose that the acceleration (measured in m/sec^2 of the car at time t is given by the formula

$$a(t) = 10e^{-t/2}.$$

(a) (5 points) Find a formula for $v(t)$, the velocity of the car (in m/sec) at time t, if $v(0) = 5$ m/sec.

(b) (5 points) Assume as in part (a) that $v(0) = 5$ m/sec, and let $s(t)$ be the position of the car (measured in meters) t seconds after the start of the race. Assume that $s(0) = 0$ meter. Where is the car after 10 seconds? (Give a numerical answer.)

Solution to (a): Since $a(t) = v'(t)$, it follows that $v(t) = \int a(t)\,dt$. Hence,

$$v(t) = \int 10e^{-t/2}\,dt = -20e^{-t/2} + C.$$

Since $v = 5$ when $t = 0$, we have $v(0) = -20 + C = 5$ (m/sec). Hence, $C = 25$. Therefore, $\boxed{v(t) = 25 - 20e^{-t/2}}$

Solution to (b): From part (a) we can write $\dfrac{ds}{dt} = 25 - 20e^{-t/2}$.

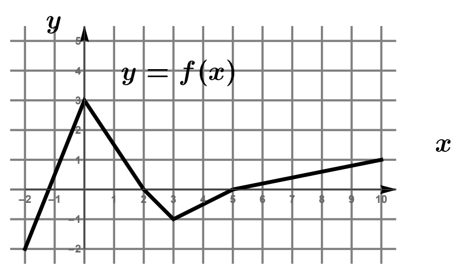
Therefore, $s(t) = \int 25 - 20e^{-t/2}\,dt = 25t + 40e^{-t/2} + C$

Since $s(0) = 40 + C = 0$, we find that $C = -40$.

Therefore, at $t = 60$ seconds (one minute), the position of the car is

$$s(60) = 25 \cdot 60 + 40e^{-60/2} - 40 \approx 1460 \text{ meters}.$$

Problem 2. The graph of the function f is shown below. Let g be the function defined by the formula $g(x) = \int_0^x f(t)\,dt$. (a) What is the value of $g(5)$? (b) What is the value of $g'(5)$?
(c) What is the value of $g''(1)$?



Solution:(a) $g(5)$ is the signed area region between the curve $y = f(x)$ and the x-axis, between $x = 0$ and $x = 5$. This is the area of a triangle of base 2 and height 3 minus the area of a triangle of base 3 and height 1, so $\boxed{g(4) = \frac{1}{2} \times 2 \times 3 - \frac{1}{2} \times 3 \times 1 = \frac{3}{2}.}$

(b) By the Fundamental Theorem of Calculus, $\boxed{g'(5) = f(5) = 0}$.

(c): $g''(1) = f'(1) =$ slope of line in the figure on the interval $[0, 2]$. So $g''(1) = \frac{0-3}{2-0} = \boxed{-\frac{3}{2}}$.

Problem 3.
(a) Evaluate the indefinite integral $\int (e^{3x} + 1)^5 e^{3x}\,dx$.

Solution: (a) Let $u = e^{3t} + 1$, then $du = 3e^{3t}dt$.

Consequently, $\int (e^{3t} + 1)^5 e^{3t}\,dt = \frac{1}{3}\int u^5\,du \frac{1}{18}u^6 + C = \boxed{\frac{1}{18}(e^{3t} + 1)^6 + C}$.

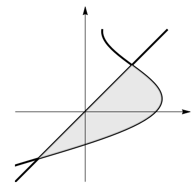
(b) Evaluate the definite integral $\int_2^3 \frac{(x-2)}{(x-2)^4 + 1}\,dx$.

Solution: Let $u = (x-2)^2$. Then $du = 2(x-2)dx$.

Hence, $\int_2^3 \frac{(x-2)}{(x-2)^4 + 1}\,dx = \frac{1}{2}\int_0^1 \frac{du}{u^2 + 1} = \frac{1}{2}\tan^{-1}(u)\Big|_0^1$

$= \frac{1}{2}(\tan^{-1}(1) - \tan^{-1}(0)) = \boxed{\pi/8}$.

Problem 4.
Find the area of the shaded region bounded by the curves $y = x$ and $x = y + 2\cos(\pi y/4)$, as shown in the figure below.

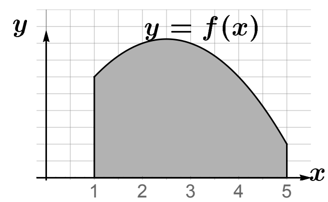


Solution: Notice that when the two curves intersect $\cos(\pi y/4) = 0$. So the curves intersect when $y = -2$ and $y = +2$. The area of the shaded region is therefore:
$\int_{-2}^{+2} \{y + 2\cos(\pi y/4)) - y\}\, dy = 2\int_{-2}^{+2} \cos(\pi y/4) dy = \frac{8}{\pi} \sin(\pi y/4)|_{-2}^{+2} = \boxed{16/\pi}$.

Problem 5.

The region pictured below is revolved about the y-axis to form a solid.
(a) Express the volume of the solid as a definite integral. (Your answer will involve the function $f(x)$.)

Solution: $\boxed{\text{Volume} = 2\pi \int_1^5 x f(x)\, dx}$



(b) Using the table of values of $f(x)$ below, approximate the volume of the solid by approximating the integral in part (a) using a midpoint sum (M_n) with $n = 4$.

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$:	24.0	29.0	32.0	33.0	32.0	29.0	24.0	17.0	18.0

Solution: Since $n = 4$, $\Delta x = \frac{(5-1)}{4} = 1$, and
$x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 5$.
Therefore $\overline{x}_1 = 1.5$, $\overline{x}_2 = 2.5$, $\overline{x}_3 = 3.5$, $\overline{x}_4 = 4.5$.
Consequently Volume $\approx M_4 = \sum_{j=1}^{4} 2\pi\overline{x}_j f(\overline{x}_j)\Delta x$
$= 2\pi(1.5 \times 29 + 2.5 \times 33 + 3.5 \times 29 + 4.5 \times 17) \times 1.0 = 2\pi \times 304 \approx \boxed{1919.09}$