2. An object moves in a straight line. Its acceleration at t seconds, in m/s^2, is given by

$$a(t) = \cos(2t).$$

The velocity of this object at $t = \pi/4$ seconds is $v(\pi/4) = 0$ m/s.

(a) (4 points) Compute the value of $\int_{\pi/4}^{\pi/2} a(t)dt$. Include **units** in your answer, and, in one brief sentence, state what this value represents, in terms of the motion of the object.

$$\int_{\pi/4}^{\pi/2} \cos(2t)\,dt = \frac{1}{2}\sin(2t)\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{1}{2}\sin(\pi) - \frac{1}{2}\sin(\pi/2)$$

$$= \boxed{-\frac{1}{2}}$$

units are in $\boxed{m/s}$ and this represents the net change in the velocity of the object from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ seconds

(b) (3 points) Compute the velocity $v(t)$ of this object at t seconds, as a function of t.

$$v(t) = \int \cos(2t)\,dt = \frac{1}{2}\sin(2t) + C$$

$$v(\pi/4) = 0 \Rightarrow \frac{1}{2} + C = 0 \Rightarrow C = -\frac{1}{2}$$

$$\therefore \boxed{v(t) = \frac{1}{2}\sin(2t) - \frac{1}{2}}$$

(c) (3 points) Compute the displacement (change in position) of the object, in meters, from 0 to $\pi/2$ seconds.

$$\Delta s = \int_0^{\pi/2} v(t)\,dt$$

$$= \int_0^{\pi/2} \frac{1}{2}\sin(2t) - \frac{1}{2} = \left(\frac{-\cos(2t)}{4} - \frac{t}{2}\right)\Big|_0^{\pi/2}$$

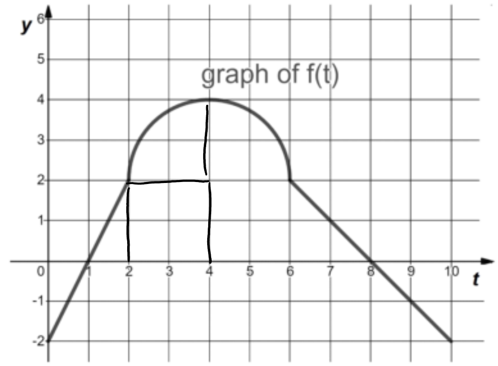
$$= \left(-\frac{1}{4}\cos(\pi) - \frac{\pi}{4}\right) - \left(\frac{-1}{4} - 0\right)$$

$$= \frac{1}{4} - \frac{\pi}{4} + \frac{1}{4} = \boxed{\frac{1}{2} - \frac{\pi}{4}} = \frac{2-\pi}{4} \text{ (meters)}$$

$$\approx -0.2854 \text{ m.}$$

3. The following is the graph of a function $y = f(t)$, for $0 \le t \le 10$, consisting of two line segments and a semicircle. We use it to define a new function, for $0 \le x \le 10$:

$$F(x) = \int_0^x f(t)\,dt$$



Indicate how you get your answers in part (a). You need not justify parts (b)-(e) below.

(a) (5 points) Calculate $F(4)$, $F'(4)$, and $f'(4)$.

$F(4) = \int_0^4 f(t)\,dt = - \quad + \quad +$

$= -\cancel{1} + \cancel{1} + 4 + \frac{\pi (2)^2}{4} = \boxed{4+\pi}$

$F'(4) = f(4) = \boxed{4}$

$f'(4) =$ slope of tangent line to graph at $t=4$ $= \boxed{0}$

(b) (1 point) At what value of x in $[0,10]$ does $F(x)$ reach its absolute maximum? At $\boxed{x=8}$

(c) (2 points) What is the absolute minimum of $F(x)$ over the interval $[0,10]$? $\boxed{\text{Min} = -1}$ (at $x = 1$)

(d) (1 point) At what value of x in $[0,10]$ does $\frac{dF}{dx}$ reach its absolute maximum value?

$\frac{dF}{dx} = f(x)$ so it's maximum is reached at $\boxed{x=4}$

(e) (1 point) On the interval $(0,2)$, is the graph of $F(x)$ concave up, concave down, neither, or cannot tell?

On $(0,2)$: $F''(x) = f'(x)$ is positive, so F is concave up.

4. (6 points) Evaluate the following integral. Show all steps.

$$\int \frac{2}{\sqrt{x}\,(2+\sqrt{x})^{41}}\,dx$$

Substitute $\boxed{u = 2+\sqrt{x}}$

$\therefore du = \frac{1}{2\sqrt{x}}dx$

$\therefore \boxed{2du = \frac{1}{\sqrt{x}}dx}$

$$= \int \frac{2}{u^{41}}\, 2\, du$$

$$= \int 4u^{-41}\, du$$

$$= 4\frac{u^{-40}}{-40} + C$$

$$= \boxed{-\frac{1}{10}(2+\sqrt{x})^{-40} + C}$$

$$= \boxed{\frac{-1}{10(2+\sqrt{x})^{40}} + C}$$

5. (4 points) Write the following limit of a Riemann Sum as a definite integral of a function over an interval.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\sin\left(1+\frac{i}{n}\right)\cos\left(1+\frac{i}{n}\right)\frac{1}{n}\right)$$

Answer is not unique.

Most natural:

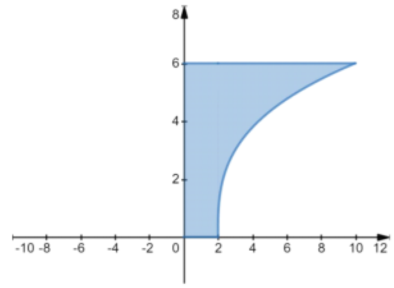
1) if $\Delta x = 1$ and we take $[a,b] = [0,1]$, then $x_i = \frac{i}{n}$ and we get $\boxed{\int_0^1 \sin(1+x)\cos(1+x)\,dx}$,

2) if $\Delta x = 1$ and $[a,b] = [1,2]$, then $x_i = 1+\frac{i}{n}$ for $i = 1, 2, \ldots, n$ so we get $\boxed{\int_1^2 \sin x \cos x\, dx}$!

6. Consider the region R shaded on the graph below. It is bounded by the following curves:

the x-axis, the y-axis, $y = 6$, and $y = 3(x-2)^{1/3}$.

$$y/3 = (x-2)^{1/3}$$
$$x-2 = (y/3)^3$$
$$\boxed{x = \frac{1}{27}y^3 + 2}$$



(a) (3 points) Set up an integral expression in terms of x equal to the area of this region. Do not evaluate.

$$A = \int_0^2 6\,dx + \int_2^{10} 6 - 3\sqrt[3]{x-2}\,dx$$

+2

+1

$$\text{OR} \quad 12 + \int_2^{10} 6 - 3\sqrt[3]{x-2}\,dx$$

$$\left[\text{OR} \quad 60 - \int_2^{10} 3\sqrt[3]{x-2}\,dx \quad \text{i.e. } 6\ \underset{10}{\square} - \square\right].$$

(b) (3 points) Set up an integral expression in terms of y equal to the area of this region. Do not evaluate.

$$A = \int_0^6 \left(\frac{1}{27}y^3 + 2\right) dy.$$

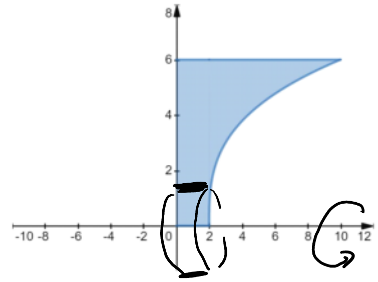
(c) (4 points) Compute the area of the region by evaluating either of your integrals above.

$$\int_0^6 \frac{1}{27}y^3 + 2\,dy = \frac{1}{27}\cdot\frac{1}{4}\cdot y^4 + 2y\Big)\Big|_0^6$$
$$= \frac{1}{3^3}\frac{1}{2^2}\cdot 6^4 + 12$$
$$= \boxed{24} \text{ square units}$$

7. Consider the same region R as in the previous problem, bounded by the curves:

the x-axis, the y-axis, $y = 6$, and $y = 3(x-2)^{1/3}$. $\Rightarrow x = 2 + \left(\frac{y}{3}\right)^3$

(a) (4 points) SET UP (do not evaluate or simplify) an integral equal to the volume of the solid of revolution obtained by rotating this region around the x-axis



Shells in y:

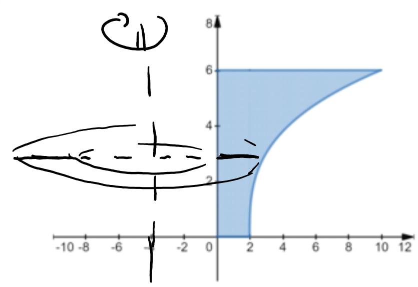
$$V_1 = \int_0^6 2\pi(\text{radius})(\text{height})\,dy$$

$$= \boxed{\int_0^6 2\pi\, y\left(\frac{1}{27}y^3 + 2\right)dy}$$

Or 2 integrals Disks/Washers in x

$$\int_0^2 \pi(6)^2\,dx + \int_2^{10} \pi(6)^2 - \pi\left(3(x-2)^{1/3}\right)^2 dx.$$

(b) (4 points) SET UP (do not evaluate or simplify) an integral equal to the volume of the solid of revolution obtained by rotating this region around the vertical line $x = -4$.



Washers in y:

$$V_2 = \int_0^6 \pi R^2 - \pi r^2\,dy$$

$$\boxed{V_2 = \int_0^6 \pi\left[\left(\frac{1}{27}y^3 + 2\right) + 4\right]^2 - \pi(4)^2\,dy.}$$

OR

Shells in x:

$$V_2 = \int_0^2 2\pi\overbrace{(x+4)}^{r}\overbrace{(6)}^{h}\,dx + \int_2^{10} 2\pi\overbrace{(x+4)}^{r}\overbrace{\left(6 - 3\sqrt[3]{x-2}\right)}^{h.}dx$$