Math 125 A . Midterm 1. Fall 2024. Instructor: Elena Pezzoli.

NAME (First,Last) : ....

UW email: ....

Student ID: ....

- Unless the problem gives you specific instructions, you can give exact answers or approximate to two decimal digits.
- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of notes (both sides). Do not share notes.
- The only calculator allowed is a TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the integral formulas in the table in the back of this page. All other integrals must be computed using the methods taught in this class and showing your work.
- Do not turn in any scratch work, or your note sheet. Only the work written on this exam will be graded.
- This exam will be scanned. Please make sure your writing is tidy and dark enough.
- This exam has 4 pages, written both sides, and 4 problems. Please make sure that your exam is complete.

Table of indefinite integrals.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1\ ; \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C; \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C; \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C; \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C; \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C; \qquad \int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$\int \tan x\,dx = \ln|\sec x| + C; \qquad \int \cot x\,dx = \ln|\sin x| + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C; \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}(\frac{x}{a}) + C$$

1. Calculate the following integrals. Show your work and write your final answer inside the box.

   (a) (5.5 points) $\int 6 \cdot x \cdot cos(x^2 + 1)\, dx$

$u = x^2 + 1$

$du = 2x \, dx$

The integral becomes

$\int 3 \cos u \, du = 3 \sin u + C$

Going beck to x

ANSWER: $\boxed{3 \sin(x^2 + 1) + C}$

(problem 1 continued)

(b) (7.5 points) $\int 5x^3\sqrt{x^2+2}\,dx$

$u = x^2 + 2 \qquad x^2 = u - 2$

$du = 2x\,dx$

The integral becomes

$$\left(\int \frac{5}{2} x^2 \sqrt{u}\,du\right) \qquad \int \frac{5}{2}(u-2)u^{1/2}\,du =$$

$$= \int \frac{5}{2}u^{\frac{3}{2}} - 5u^{\frac{1}{2}}\,du = \frac{5}{2}\frac{u^{5/2}}{\frac{5}{2}} - 5\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= u^{5/2} - \frac{10}{3}u^{3/2} + C$$

going back to x

ANSWER: $\boxed{(x^2+2)^{5/2} - \frac{10}{3}(x^2+2)^{3/2} + C}$

(problem 1 continued)

(c) (5 points) $\int_0^{\frac{\pi}{4}} \tan(\theta) \cdot \cos^2(\theta)\, d\theta$ $= \int_0^{\pi/4} \sin\theta \cdot \cos\theta \; d\theta$

$u = \sin\theta$

$du = \cos\theta \; d\theta$

$$\int_{\sin 0}^{\sin \frac{\pi}{4}} u \; du = \frac{u^2}{2} \Big|_0^{1/\sqrt{2}} = \frac{1}{4}$$

ANSWER: (Give an exact answer,simplifying as much as possible) : $\boxed{\frac{1}{4}}$

2. Let $f(x) = \int_{x^2}^{x^3} \sin(t^2)dt$.

(a) (5 points) Calculate $\frac{df}{dx}(\sqrt{2})$

$$f(x) = -\int_a^{x^2} \sin(t^2)\,dt + \int_a^{x^3} \sin(t^2)\,dt$$

$$f'(x) = -2x \sin(x^4) + 3x^2 \sin(x^6)$$

$$f'(\sqrt{2}) = -2\sqrt{2} \sin(4) + 6 \sin(8)$$

ANSWER (approximate to two decimal digits): $\boxed{8.08}$

(b) (2 points) Is f INCREASING or DECREASING at $x = \sqrt{2}$? (CIRCLE THE CORRECT ANSWER). In once sentence explain why.

Because $f'(\sqrt{2}) > 0$

(c) (5 points) Fill the THREE boxes in the expression below to write a Riemann sum with n subdivisions, taking the sample points to be rightpoints (i.e. a RIGHT-HAND SUM) that approximates $f(2)$. Do not evaluate the sum.

$$f(2) = \int_4^8 \sin(t^2)\,dt$$

4

$$\sum_{i=\boxed{1}}^{\boxed{n}} \boxed{\sin\left(\left(4 + 4\frac{i}{n}\right)^2\right) \cdot \frac{4}{n}}$$

$$\Delta x = \frac{8-4}{n} = \frac{4}{n} \qquad x_i = 4 + i \cdot \frac{4}{n}$$

3. The acceleration of an object traveling along a straight line is $a(t) = 2t - 3$ cm/sec^2. At time $t = 1$ the velocity of the object is $v(1) = 0$ cm/sec

a) (2 points) Find a formula for the velocity of the object at time t.

$$v(t) = t^2 - 3t + C$$
$$2 = 0 + C \qquad C = 2$$

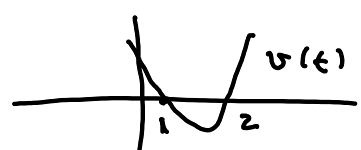
ANSWER: $\boxed{v(t) = t^2 - 3t + 2}$ cm/sec

(b) (2 points) Write one integral that calculates the displacement of the object in the time interval from $t = 1$ to $t = 3$. You do not need to evaluate the integral.

ANSWER: $\boxed{\int_1^3 t^2 - 3t + 2 \, dt}$ cm

(c) (6 points) Calculate the distance travelled by the object in the time interval from $t = 1$ to $t = 3$.

$$\int_1^3 |t^2 - 3t + 2| \, dt =$$



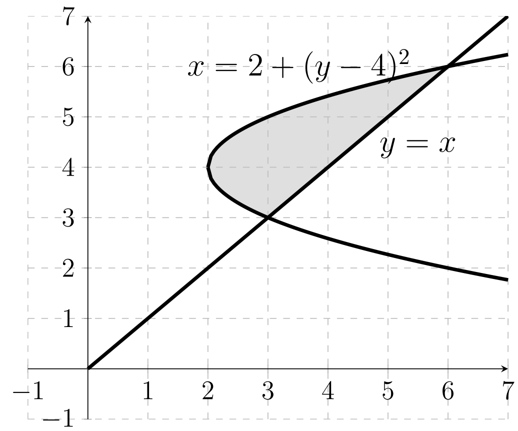
$$= \int_1^2 -t^2 + 3t - 2 \, dt + \int_2^3 t^2 - 3t + 2 \, dt =$$

$$= -\frac{t^3}{3} + \frac{3}{2}t^2 - 2t \Big|_1^2 + \frac{t^3}{3} - \frac{3t^2}{2} + 2t \Big|_2^3 =$$

$$\left(-\frac{8}{3} + 6 - 4\right) - \left(-\frac{1}{3} + \frac{3}{2} - 2\right) +$$
$$\left(9 - \frac{27}{2} + 6\right) - \left(\frac{8}{3} - 6 + 4\right) =$$
$$=$$

ANSWER: $\boxed{1}$ cm

4. This problem asks you to calculate the area of the region below bounded by the curves $y = x$ and $x = 2 + (y-4)^2$:



a) (5 points) In the box below write an integral (or integrals) that compute the area of the region.

ANSWER: $\int_3^6 y - 2 - (y-4)^2 \, dy =$

b) (5 points) Calculate the integral(s).

$$= \frac{y^2}{2} - 2y - \frac{(y-4)^3}{3} \Big|_3^6$$

$$= 18 - 12 - \frac{8}{3} - \frac{9}{2} + 6 - \frac{1}{3} =$$

$12 - 3 - 4.5$

ANSWER: 4.5