Your Name

Your Signature

Student number

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed books. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of handwritten notes (you can write on both sides).
- You can use only Texas Instruments TI-30X calculator.
- Give your answers in exact form, not decimals, unless instructed otherwise.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- **Check your work carefully.** We will award only limited partial credit.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Make sure that your exam is complete.

1. (10 points) Estimate the net area between the graph of the function $f(x) = 1/(\ln x)$ and the x-axis, between $x = 2$ and $x = 4$. Use the midpoint rule with 3 subintervals. Give your answer as a decimal number with 5 significant digits.

Solution:

We have $\Delta x = (4-2)/3 = 2/3$. The midpoints of the three intervals are $2\frac{1}{3} = 7/3, 3$ and $3\frac{2}{3} = 11/3$. The formula for the midpoint approximation M_n with $n = 3$ is

$$M_3 = \frac{2}{3} \cdot \left(\frac{1}{\ln(7/3)} + \frac{1}{\ln(3)} + \frac{1}{\ln(11/3)} \right) \approx \boxed{1.90674}.$$

2. Compute the integrals.

   (a) (4 points)

$$\int \tan^5 x \ \sec^2 x \, dx$$

   (b) (6 points)

$$\int x \left[\tan\left(5x^2+8\right) \sec\left(5x^2+8\right)\right]^2 dx$$

Solution:

(a) Substitution: $u = \tan x$, $du = \sec^2 x dx$.

$$\int \tan^5 x \sec^2 x dx = \int u^5 du = \frac{1}{6}u^6 + C = \boxed{\frac{1}{6}\tan^6 x + C}$$

(b)

$$\int x \left(\tan\left(5x^2+8\right) \sec\left(5x^2+8\right)\right)^2 dx = \int \tan^2\left(5x^2+8\right)\sec^2\left(5x^2+8\right) x dx$$

Substitution: $u = \tan\left(5x^2+8\right)$, $du = \sec^2\left(5x^2+8\right) 10x dx$. Hence, $(1/10)du = \sec^2\left(5x^2+8\right) x dx$.

$$\int \tan^2\left(5x^2+8\right)\sec^2\left(5x^2+8\right) x dx = \int u^2 (1/10) du = \frac{1}{30}u^3 + C = \boxed{\frac{1}{30}\tan^3\left(5x^2+8\right) + C}$$

3. (10 points) A group of scientists studying climate change proposed the following model for the global temperatures. Starting January 1, 2023, the temperature will rise at the rate of $0.05+0.01t$ degrees per year, for 5 years. For 5 years starting January 1, 2028, the temperature will change at the rate of $0.1-0.01(t-5)^2$ degrees per year. In both formulas, $t=0$ represents January 1, 2023. Find the global temperature increase between January 1, 2023 and January 1, 2033, according to the model.

   Give your answer as a decimal number with 5 significant digits.

Solution:

We will apply the net change formula. The temperature increase will be

$$\int_0^5 (0.05+0.01t)dt + \int_5^{10} (0.1-0.01(t-5)^2)dt = \int_0^5 (0.05+0.01t)dt + \int_5^{10} (0.1-0.01(t^2-10t+25))dt$$

$$= (0.05t+0.01t^2/2)\Big|_0^5 + (0.1t-0.01(t^3/3-10t^2/2+25t))\Big|_5^{10} \approx \boxed{0.458333}$$

4. (10 points) Find $f'(-2)$ if

$$f(x) = \int_{x+1}^{x^2+x} t\, e^{2t+1} dt.$$

Give the answer in exact simplified form.

Solution:

$$\begin{aligned} f'(x) &= (x^2+x)e^{2(x^2+x)+1}(2x+1) - (x+1)e^{2(x+1)+1}, \\ f'(-2) &= ((-2)^2+(-2))e^{2((-2)^2+(-2))+1}(2(-2)+1) - ((-2)+1)e^{2((-2)+1)+1} \\ &= 2e^5(-3) + e^{-1} = \boxed{-6e^5 + 1/e} \approx -890.111 \end{aligned}$$

5. (10 points) In a physics experiment, an object moving along a straight line was given the acceleration $a(t) = t^{-2} + 1$ between times $t = 1/4$ and $t = 10$. The object was moving with the initial speed $v(1/4) = -3.75$ at time $t = 1/4$.

   Give answers in the exact form or in the decimal form with at least 5 significant digits.

   (a) (4 points) Find the displacement of the object between the times $t = 1/2$ and $t = 2$.

   (b) (6 points) Find the total distance traveled over the time interval $(1/3, 5)$.

Solution: Notation: $a(t)$ – acceleration, $v(t)$ – velocity, $s(t)$ – position.

$$a(t) = t^{-2} + 1,$$
$$v(t) = \int a(t)dt = -t^{-1} + t + C_1.$$

Since $v(1/4) = -3.75$, we have $-(1/4)^{-1} + (1/4) + C_1 = -3.75$ and, therefore, $C_1 = 0$.

$$v(t) = -t^{-1} + t,$$
$$s(t) = \int v(t)dt = -\ln|t| + t^2/2 + C_2.$$

(a)

$$s(2) - s(1/2) = -\ln 2 + 2^2/2 + C_2 - (-\ln(1/2) + (1/2)^2/2 + C_2)$$
$$= \boxed{-2\ln 2 + 15/8} \approx 0.488706$$

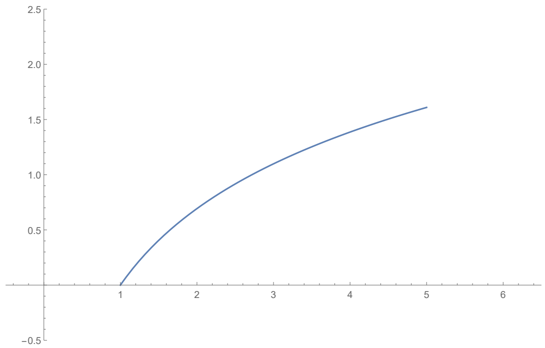
(b) We will find the intervals where v was positive and negative. First we solve $v(t) = 0$.

$$v(t) = 0; \qquad -t^{-1} + t = 0; \qquad t = 1/t; \qquad t = 1.$$

Velocity was negative between 1/3 and 1. It was positive between 1 and 5. The total distance traveled was

$$|s(1) - s(1/3)| + |s(5) - s(1)|$$
$$= |-\ln 1 + 1^2/2 + C_2 - (-\ln(1/3) + (1/3)^2/2 + C_2)| + |-\ln 5 + 5^2/2 + C_2 - (-\ln 1 + 1^2/2 + C_2)|$$
$$= \boxed{\ln 3 - \ln 5 + 104/9} \approx 11.0447$$

6. A region A lies above the x-axis, below the graph of $y = \ln x$, between $x = 1$ and $x = 5$ (see the figure below).



Find the volume of the solid of revolution obtained by rotating the region A about the line $x = 6$.

Give the answer in the exact form or in the decimal form with at least 5 significant digits.

Solution:

Since $y = \ln x$, we have $x = e^y$. We cut the the body of revolution into horizontal annuli with radii $6 - x = 6 - e^y$ and 1. The area of an annulus is

$$\pi((6-e^y)^2 - 1^2) = \pi(36 - 12e^y + e^{2y} - 1) = \pi(35 - 12e^y + e^{2y}).$$

The top of the object is at the height $\ln 5$. The volume is equal to

$$\int_0^{\ln 5} \pi(35 - 12e^y + e^{2y})dy = \pi(35y - 12e^y + e^{2y}/2)\Big|_0^{\ln 5} = \pi(35\ln 5 - 12\cdot 5 + 5^2/2 + 12 - 1/2)$$
$$= \boxed{\pi(35\ln 5 - 36)} \approx 63.8696$$