Your Name

Your Student ID Number

Professor's Name

Lecture Section (circle one)

A (8:30), B (9:30), C (12:30), D (1:30)
E (hybrid)

- Turn off and stow away all cell phones, watches, headphones, and any other connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive full credit, you must **show complete, correct, and readable work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Write your final answer in the "Answer: _____" area for each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3} + \frac{1}{7}$).
- All exam pages are double-sided except for this cover page and the last blank page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, make sure that your exam is complete. Good luck!

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

1. Evaluate the following integrals. Show your work.

   (a) (6 points) $\int_0^1 \frac{x}{x^2+3x+2}dx$

   Answer: _____

   (b) (6 points) $\int_0^2 \frac{x}{x^4+2x^2+2}dx$

   Answer: _____

2. Evaluate the following integrals. Show your work.

(a) (6 points) $\displaystyle\int \frac{e^x}{\sqrt{4+e^{2x}}}\,dx$

Answer: _____

(b) (6 points) $\displaystyle\int \sec^3(x)\,dx$ (*hint: try integration by parts*)

Answer: _____

3. (8 points) For each integral below, state which method applies best.

   Your answer should be in one of the following forms:

   - u-substitution, with $u =$... (specify the substitution)
   - integration by parts, with $u =$..., and $dv =$ ... (specify the parts)
   - trigonometric substitution, with $x =$... (specify the trig sub)
   - partial fractions, with fractions: $\frac{A}{(\ldots)} + \ldots$ (specify the fractions, do not calculate A, etc)

   No need to justify or compute anything – and do not evaluate the integrals!

   (a) $\int \sin(x)\cos^2(x)\,dx$ Method:

   (b) $\int x\sec^2(x)\,dx$ Method:

   (c) $\int \frac{2x+1}{x^4+x^3+x^2}\,dx$ Method:

   (d) $\int \frac{x^2}{(x^2-4)^{3/2}}\,dx$ Method:

4. A drone starts from a height of 2 m above the ground at $t = 0$, and flies straight up, with velocity at t seconds, $0 \leq t \leq 10$, given by

$$v_1(t) = 0.3t^2 \; m/s.$$

At $t = 10$ seconds, its battery fails, so from that time on the acceleration acting on the drone is

$$a_2(t) = -9.8 \; m/s^2,$$

causing it to eventually fall down and crash on the ground.

(a) (5 points) What is the drone's height above ground at the moment when its battery fails?

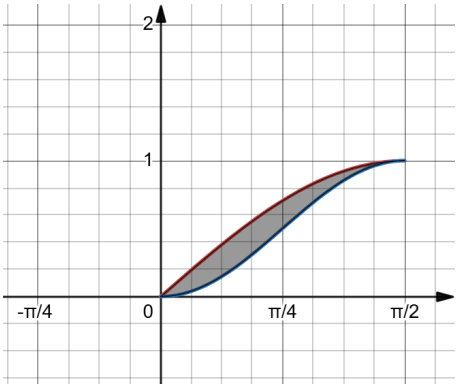
Answer: _____ meters

(b) (5 points) How long does the drone take to crash on the ground, from the time its battery fails? Round your answer to the nearest tenth of a second.

Answer: _____ seconds

5. (8 points) Find the area bounded by the curves:

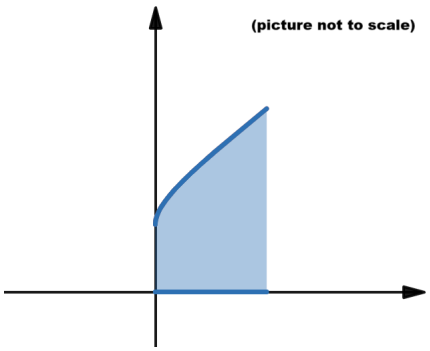
$$y = \sin x \text{ and } y = \sin^2 x, \text{ for } 0 \leq x \leq \pi/2.$$



Answer: Area = _____

6. Let R be the region in the xy-plane bounded by the curves:

$$y = e^{\sqrt{x}},\ x = 0,\ x = 2 \text{ and } y = 0.$$



(a) (5 points) **Set up, but do not evaluate,** an integral (or a sum of integrals) equal to the volume of the solid of revolution obtained by revolving R about the vertical line $x = 3$ using the **method of cylindrical shells**.

Answer: _____

(b) (5 points) **Set up, but do not evaluate,** an integral (or a sum of integrals) equal to the volume of the solid of revolution obtained by revolving R about the vertical line $x = -1$ using the **method of disks and washers**.

Answer: _____

7. (10 points) Find the mass of an empty leaky bucket given the following information:

   - The bucket held 10 liters of water at the ground level and was lifted to the height of 16 meters.
   - While being lifted, water leaked out of the bucket at the rate of 1/8 liters per meter.
   - The total work done was 1960 J.

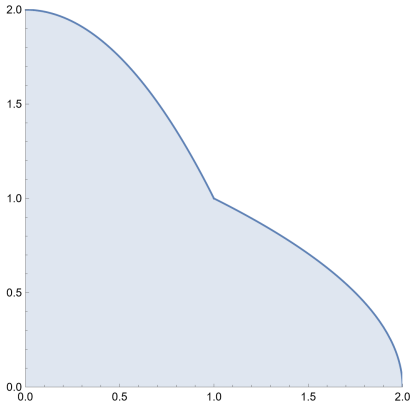
   Assume that the mass of the rope used to lift the bucket was negligible and can be ignored.
   The mass of 1 liter of water is 1 kilogram, and the acceleration due to gravity is 9.8 meters/sec^2.

ANSWER: mass of bucket $m =$ _____ kg

8. (10 points) Consider the region in the first quadrant bounded below by the x-axis, bounded on the left by the y-axis, bounded above by the graph of $y = 2 - x^2$ for $0 \leq x \leq 1$, and bounded above by the graph of $y = \sqrt{2-x}$ for $1 \leq x \leq 2$. See the picture.

   Find the center of mass. Give the answer in the exact form.

   Note: You can use symmetry but to get full credit, you have to provide a computation supporting the use of symmetry. Saying that the picture looks symmetric is not enough.



ANSWER: $(\bar{x}, \bar{y})$ = _____

9. (10 points) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dt} = 1 + t^2 + y^2 + t^2 y^2, \quad y(0) = 1.$$

For full credit, write your answer in explicit form, $y = f(t)$, and simplify it.

ANSWER: $y =$ _____

10. A pan of lasagna has an internal temperature of 40° F at the time when it is placed in an oven whose temperature is kept constant at 380° F.

    (a) (3 points) Newton's Law of Cooling states that the rate of cooling (or heating) of an object is proportional to the temperature difference between the object and its surroundings. Use this to write a differential equation and an initial condition for the internal temperature $y(t)$ of the lasagna at t hours after it was placed in the oven. Your equation should involve an unknown proportionality constant k.

    Differential Equation: _____ and $y(0) =$ _____

    (b) (5 points) After a half hour in the oven, the internal temperature of the lasagna reaches 140° F. Compute the constant k and find the temperature y as a function of time t, in hours.

    Answer: $y(t) =$ _____

    (c) (2 points) The lasagna is finished baking when its internal temperature reaches 165° F. When will this happen?

    Answer: At $t =$ _____ hours

    (round to the nearest two decimals)

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.