Your Name

Student ID

Professor's Name

- Turn off and stow away all cell phones, smart watches, music players, and any other smart or connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). No other papers or notebooks are allowed. Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \dfrac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \dfrac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right), \quad a > 0$

- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\dfrac{\pi}{3}$ or $5\sqrt{3}$).
- All pages are double-sided except for this cover page and the last pages. You may use the blank pages for extra room but, if you want us to grade these spare pages, clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank page(s) at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, check that your exam is complete. Good luck!

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) (5 points) $\int \left(\frac{1}{x-x^2} - \frac{2x}{x-x^2} \right) dx$

   (b) (5 points) $\int \cos(2x) e^x \, dx$

2. Evaluate the following integrals. Show all work. Simplify and box your answers.

(a) (5 points) $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$

(b) (5 points) $\displaystyle\int \frac{dx}{x\sqrt{x+4}}$

3. A particle moves along a straight line with velocity $v(t) = 3\cos(2t)$.

   (a) (5 points) Compute the particle's displacement from $t = 0$ to $t = 3\pi/4$.

   (b) (5 points) Compute the total distance that the particle travels from $t = 0$ to $t = 3\pi/4$.

4. (10 points) Determine the arc length of the curve:

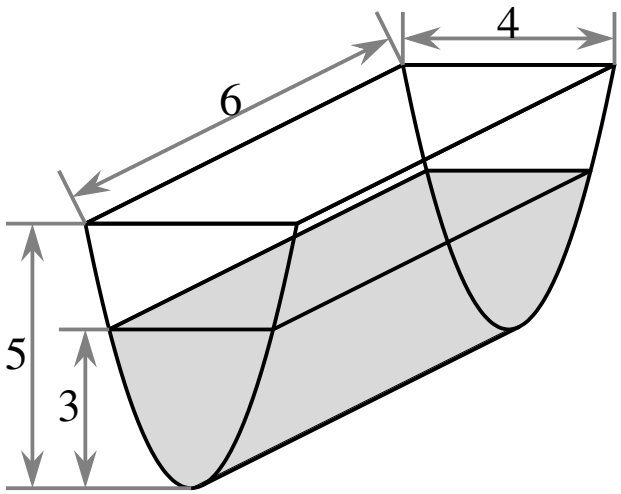
$$y = \frac{x^3}{3} + \frac{1}{4x} - 5$$

from $x = \frac{1}{4}$ to $x = 1$. Give the answer in exact simplified form.

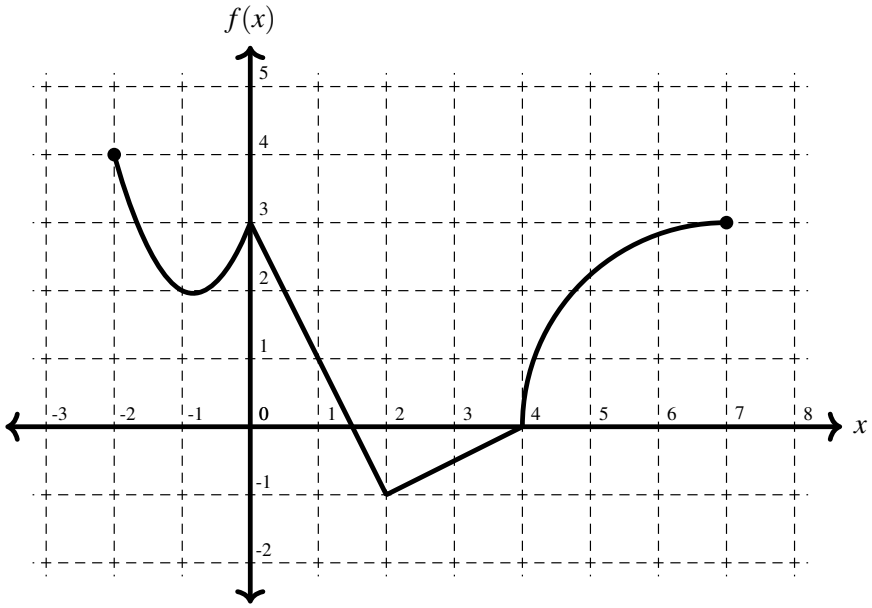
5. (10 points) The tank shown below (all dimensions are in meters) has parabolic ends and vertical cross-sections. The tank is filled with water to a depth of 3 meters. Suppose $\rho = 1000$ kg/m^3 is the density of water and $g = 9.8$ m/s^2 is the acceleration due to gravity.

   Give an integral expressing the work done in pumping all of the water to the top of the tank.

   Do NOT evaluate the integral. Just set it up.



6. (10 points) Here's the graph of a function $f(x)$, consisting of a portion of a parabola, two line segments, and a quarter circle. Use it to answer the questions below. You may use well-known formulas for areas of geometric objects.



(a) (4 points) Compute $\lim_{n\to\infty} \sum_{i=0}^{n} \left(f\left(\frac{7i}{n}\right) \frac{7}{n} \right)$.

(b) (4 points) Let $g(x) = \int_{\sin(\pi x)}^{5} f(t)\,dt$. Find $g'\left(\frac{1}{6}\right)$.

(c) (2 points) The Trapezoidal Rule approximation T_3 of $\int_{-2}^{0} f(x)\,dx$ is (circle one):

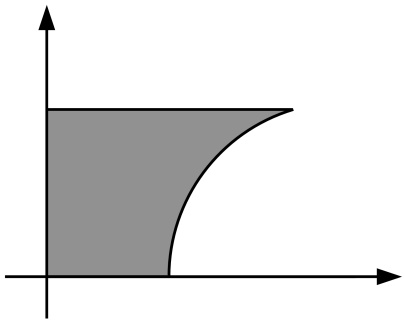
(i) an underestimate or (ii) an overestimate

because the function is (circle one):

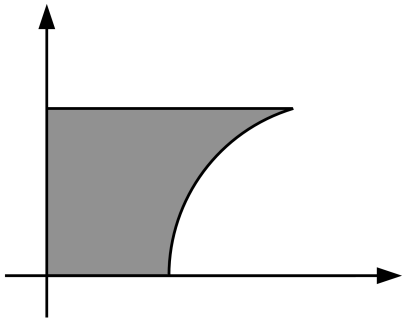
(a) positive (b) negative (c) increasing (d) decreasing (e) concave up (f) concave down

7. (10 points) Let $\mathcal{R}$ be the region in the first quadrant bounded by $y = \sqrt{\ln(x)}$, $y = 2$, and the two axes. Let $\mathcal{S}$ be the solid formed by revolving $\mathcal{R}$ about the x-axis. (Pictures are not to scale.)

   (a) (3 points) Set up an integral expression to evaluate the volume of $\mathcal{S}$ using the **washer method**. Do NOT evaluate the integral expression. Just set it up.



   (b) (3 points) Set up an integral expression to evaluate the volume of $\mathcal{S}$ using the **shell method**. Do NOT evaluate the integral expression. Just set it up.



   (c) (4 points) Find the volume of $\mathcal{S}$. (Use whichever method from parts (a) or (b) you prefer.)

8. (10 points) Let R be the region bounded by the curves $y = 4\sin x$ and $y = 2\sin x$ and between $x = 0$ and $x = \pi$. By symmetry, we can tell that the x-coordinate of the centroid (center of mass) of R is $\pi/2$. Find the y-coordinate of the centroid of R.

9. (10 points) Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{y+3}{x \ln x}$$

that satisfies the initial condition:

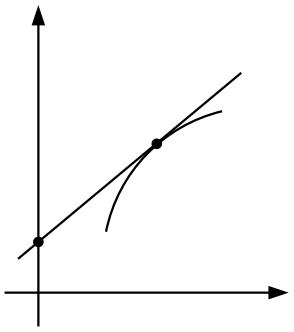
$$y(e^5) = 27.$$

For full credit, write your answer in explicit form, $y = f(x)$.

10. (10 points) At all $x > 0$, a certain curve $y = f(x)$ has the following property:

The tangent line to the curve $y = f(x)$ at any point (a, b) on the curve intersects the y-axis at $(0, \sqrt{b})$.

(a) (5 points) Find a differential equation satisfied by $f(x)$.



(b) (5 points) Suppose the curve passes through the point $(1, 16)$. Find the function $f(x)$ for $x > 0$ by solving the differential equation you found in (a).

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