Math 125 **Final Examination** **Winter 2016**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use only the Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give all answers in EXACT form (like $\pi/4$ or $1+\sqrt{2}$) unless explicitly given directions otherwise.
- You may use any of the 20 integrals in the table on p. 495 of the text without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Evaluate the following indefinite integrals.

   (a) (5 points) $\int \sin(x)\sqrt{\cos(x)}\,dx$

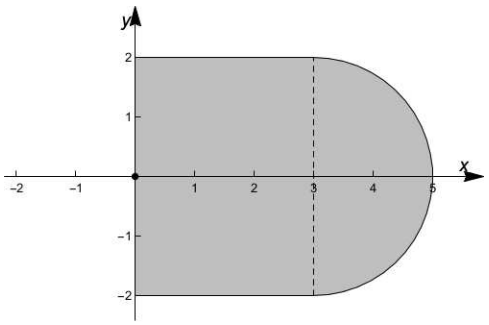
   (b) (5 points) $\int \sqrt{3-2x-x^2}\,dx$

2. Evaluate the following definite integrals.

   (a) (5 points) $\displaystyle\int_0^{\pi} \sec\left(\frac{x}{3}\right) \tan^3\left(\frac{x}{3}\right)\, dx$

   (b) (5 points) $\displaystyle\int_{-1}^{2} \frac{x}{x^2+2x+10}\, dx$

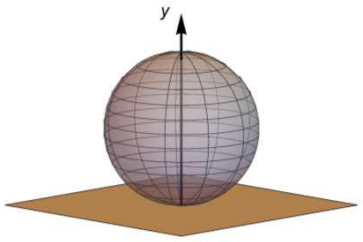
3. (10 points) Consider the region in the xy-plane formed by a rectangle of height 4 and width 3 and a half-disk of radius 2 centered at $(3,0)$, as shown in the figure. Compute $\overline{x}$, the x-component of the centroid of the region.



4. (a) (5 points) Does the improper integral $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$ converge? If yes, to what?

(b) (5 points) Does the improper integral $\displaystyle\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx$ converge? If yes, to what?

5. (10 points) A spherical tank of radius R meters is resting on the ground; so its center is located R meters above ground level (see figure). The tank is initially empty, and water is pumped from ground level into the tank until the tank is half full. Find the work (in Joules) required to do this.
   Note: Water has a mass density of 1000 kilograms per cubic meter and the acceleration due to gravity is 9.8 meters/sec^2.



6. Consider the curve $y = 2x^{5/2}$.

   (a) (4 points) Set up a definite integral for the arc length of this curve from $x = 1$ to $x = 4$. *DO NOT EVALUATE THE INTEGRAL.*

   (b) (6 points) Use the Trapezoid Rule with $n = 4$ subintervals to estimate the integral in part (a). Leave your answer in exact form.

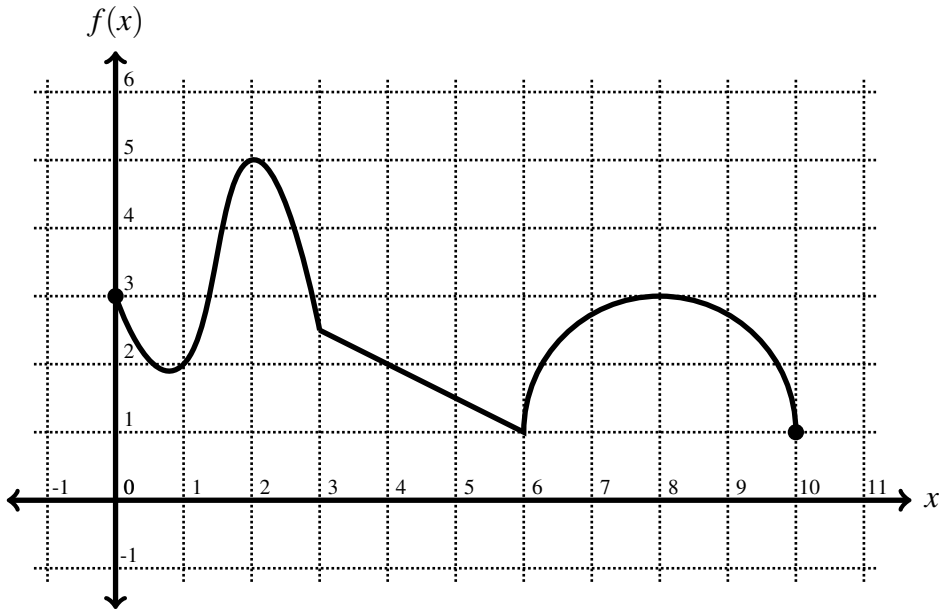
7. Consider the region $\mathscr{R}$ above the x-axis and under the curve

$$y = \sin^2 x \qquad \text{for} \qquad 0 \leq x \leq \pi/2.$$

(a) (4 points) Find the area of this region $\mathscr{R}$.

(b) (6 points) The region $\mathscr{R}$ is rotated about the y-axis. Find the volume of the resulting solid.

8. The graph of $f(x)$ is shown below. Use it to answer the following questions.



(a) (4 points) Compute the average value of $f(x)$ on the interval $[4,10]$.

(b) (6 points) Let $g(x) = \int_{x^2}^{7} f(t)\,dt$. Calculate $g''(2)$.

9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{x\cos(x^2)}{y}, \quad y(0) = -3.$$

Give your answer in the form $y = f(x)$.

10. (10 points) Among the 30 thousand students of the University of Washington, the rate of the spread of the app *Sbreak* is proportional to the product of the number of students P (in thousands) who have the app on their phones and the number of students $30 - P$ who do not (again in thousands). That is,

$$\frac{dP}{dt} = kP(30 - P)$$

where k is a positive proportionality constant, P is in thousand of students and t is in hours.

Initially, one thousand students have the app on their phones. In 4 hours, 5 thousand students have the app on their phones. According to this model, when will half of the students have the app on their phones?