This exam is closed book. You may use one 8.5" × 11" sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.

Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.

A scientific calculator is allowed, but graphing calculators are not allowed.

In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

You may use any of the 20 integrals in the table on p. 484 of the text (p. 506 if you have the 5th edition of Stewart) without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.

Place a box around your answer to each question.

If you need more room, use the backs of the pages and indicate that you have done so.

Raise your hand if you have a question.

This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (12 total points) Evaluate the following indefinite integrals.

(a) (6 points) \[ \int 2x \ln(x + 5) \, dx \]

(b) (6 points) \[ \int x^3 \sqrt{9 + 4x^2} \, dx \]
2. (12 total points) Evaluate the following definite integrals.

(a) (6 points) \[ \int_{1}^{3} \frac{1}{x^2 + x^3} \, dx \]

(b) (6 points) \[ \int_{3}^{9/2} \frac{x - 2}{\sqrt{6x - x^2}} \, dx \]
3. (10 total points) Let $\mathcal{R}$ be the region bounded on top by $y = \frac{1}{\sqrt{1+x^2}}$, on the bottom by the $x$-axis, on the left by the $y$-axis, and on the right by the line $x = B$.

(a) (3 points) Find (in terms of $B$) the volume of the solid obtained by rotating $\mathcal{R}$ about the $x$-axis.

(b) (2 points) Does the volume in part (a) have a finite limit as $B \to \infty$? If so, what is that limit?

(c) (3 points) Find (in terms of $B$) the volume of the solid obtained by rotating $\mathcal{R}$ about the $y$-axis.

(d) (2 points) Does the volume in part (c) have a finite limit as $B \to \infty$? If so, what is that limit?
4. (12 total points) Xander, an indestructible robot, has jet boots that provide enough force to give him a vertical acceleration of $10.3 \, \text{m/s}^2$ (if there were no gravity), and they have enough fuel to do so for 8 seconds. Suppose he uses them to lift himself straight up, starting at rest on the ground. Because the acceleration due to gravity is $9.8 \, \text{m/s}^2$ in the downward direction, his net acceleration in the positive vertical direction during these 8 seconds is $10.3 - 9.8 = 0.5 \, \text{m/s}^2$.

(a) (5 points) At what height above the ground will Xander run out of fuel?

(b) (2 points) What is his velocity when he runs out of fuel?

(c) (5 points) After he runs out of fuel, his acceleration is the acceleration due to gravity. How long after he runs out of fuel will he hit the ground? Give your answer (in seconds) in decimal form, correct to at least the second digit after the decimal point.
5. (10 points) Let \( R \) be the region bounded by the \( x \)-axis, the \( y \)-axis, the line \( y = 2 \), and the curve \( y = \sqrt{x - 1} \). A container is built in the shape of the solid of revolution formed by rotating \( R \) about the \( y \)-axis, and the container is filled with cold-pressed olive oil. Find the work done in pumping all of the olive oil to the top of the container. (Units on the \( x \)- and \( y \)-axes are meters, the density of cold-pressed olive oil is 915 kg/m\(^3\), and the acceleration due to gravity is 9.8 m/s\(^2\).)

Give your answer (in joules = kg \cdot m/s\(^2\) \cdot m) in decimal form, correct to the nearest joule.
6. (10 total points) Let $R$ be the region bounded on top by the line $y = 1$, on the left by the $y$-axis, and on the right by the curve $x = g(y)$ for $0 \leq y \leq 1$, where $g(y)$ is a continuous function with $g(0) = 0$ and $g(y) \geq 0$ for $0 \leq y \leq 1$. No formula is known for $g(y)$, but we do have the following table of values:

<table>
<thead>
<tr>
<th>$y$</th>
<th>0</th>
<th>0.25</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = g(y)$</td>
<td>0</td>
<td>0.70</td>
<td>1.30</td>
<td>1.70</td>
<td>2.00</td>
</tr>
</tbody>
</table>

(a) (3 points) Set up an integral (in terms of $g(y)$) for the volume of the solid obtained by rotating the region $R$ about the $x$-axis.

(b) (7 points) Use the trapezoid rule with $n = 4$ subintervals to estimate the integral in part (a). Give your answer in decimal form, correct to at least the second digit after the decimal point.
7. (12 points) Find the $x$- and $y$-coordinates of the center of mass of a uniform plate bounded on top by $y = \cos x$ for $0 \leq x \leq \pi$, on the bottom by the line $y = -1$, and on the left by the $y$-axis. Give your answers in exact form.
8. (10 points) Find the solution of the differential equation

\[ \frac{dy}{dx} = \frac{y^2 - 2y}{x} \]

that satisfies the condition \( y(1) = 1 \). Solve for \( y \), giving your answer in the form \( y = f(x) \).
9. (12 total points) Your friend wins the lottery, and gives you $P_0$ dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously (a pretty good approximation to reality if you make regular frequent withdrawals) at a rate of $3600$ per year.

(a) (3 points) Set up a differential equation for the amount of money $P(t)$ (in dollars) left in the account at time $t$ (in years).

(b) (3 points) Solve the differential equation to obtain a formula for $P(t)$. Your formula should involve $P_0$.

(c) (3 points) If $P_0 = 20,000$, then how much money is left in the account 4 years later?
Give your answer (in dollars) in decimal form, correct to the nearest cent (i.e., nearest $0.01$).

(d) (3 points) What should $P_0$ be so that after 4 years your account has exactly $0$ left over?
Give your answer (in dollars) in decimal form, correct to the nearest cent (i.e., nearest $0.01$).