

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- This exam is closed book. You may use one  $8\frac{1}{2}'' \times 11''$  sheet of handwritten notes (both sides). Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use any of the 20 integrals on p. 484 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	8	
4	12	
5	8	
6	10	

Question	Points	Score
7	8	
8	8	
9	10	
10	12	
Total	100	

1. (12 total points) Evaluate the following integrals.

(a) (6 points)  $\int \ln(1 + \sqrt{x}) dx$

(b) (6 points)  $\int \frac{x^2 + 2x + 2}{(x+1)^3} dx$

2. (12 total points) Evaluate the following integrals.

(a) (6 points)  $\int \frac{dx}{x + 2\sqrt{x+3}}$

(b) (6 points)  $\int \frac{x}{\sqrt{6x-x^2}} dx$

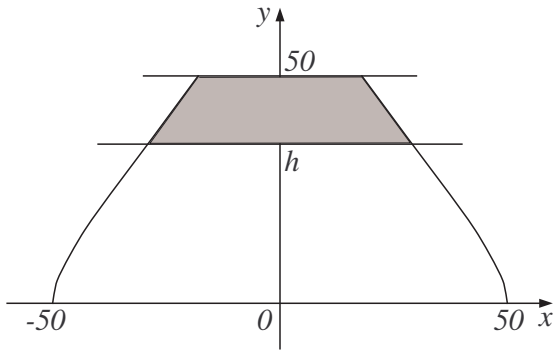
3. (8 points) Consider the improper integral

$$\int_1^{\infty} \frac{1}{x+x^2} dx.$$

Evaluate this integral or explain why it does not converge.

4. (12 total points) A shape  $S$  is bounded by the  $x$ -axis, the line  $y = 50$ , the curve  $x = 50e^{-(y/50)^2}$ , and the curve  $x = -50e^{-(y/50)^2}$ . A barrier comes down and covers the shape  $S$  between height  $h$  and height 50.

- (a) (3 points) Express the area not covered by the barrier (the unshaded area) in terms of an integral.

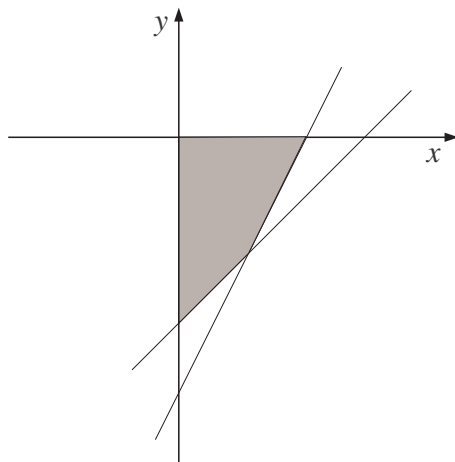


- (b) (4 points) Suppose that the horizontal line  $y = h$  at the bottom of the barrier starts at the top with zero velocity at time  $t = 0$  and descends with acceleration  $a(t) = -6t$ . Find a formula for  $h$  in terms of  $t$ .

- (c) (5 points) If the barrier descends as in part b), find a formula in terms of  $t$  for the rate of change of the area not covered by the barrier.

5. (8 points) Let  $R$  be the region bounded by the lines  $x = 0$ ,  $x = \pi/2$ , and  $y = 3$ , and by the curve  $y = 1 + \sin x$ . Let  $S$  be the solid obtained by rotating  $R$  about the line  $y = 3$ . Compute the volume of  $S$ .

6. (10 points) The shaded region shown below is bounded by the  $y$ -axis, the  $x$ -axis, the line  $y = x - 8$ , and the line  $y = 2x - 11$ . It is revolved about the  $y$ -axis to create a tank, with units in meters. The tank is filled with a liquid with density  $800 \text{ kg/m}^3$ . Express the work required to pump all of the liquid out over the top of the tank in terms of integrals, but do not evaluate these integrals.



7. (8 points) Let  $R$  be the region bounded by the curves  $y = 4 \sin x$  and  $y = 2 \sin x$  and between  $x = 0$  and  $x = \pi$ . By symmetry, we can tell that the  $x$ -coordinate of the centroid of  $R$  is  $\pi/2$ . Find the  $y$ -coordinate of the centroid of  $R$ .



8. (8 total points) Let  $L$  be the arclength of the curve  $y = \tan x$  between the origin and  $(\frac{\pi}{3}, \sqrt{3})$ .

(a) (4 points) Express  $L$  as a definite integral.

(b) (4 points) Use Simpson's rule with  $n = 4$  subdivisions to estimate  $L$ . Give a decimal answer.

9. (10 points) Find the solution of the differential equation

$$y' \cos^2 x = 2 + y$$

that satisfies  $y(\pi/4) = 0$ .

10. (12 points) Two streams flow into a lake. One stream brings in clean water at the rate of 0.2 MCM (millions of cubic meters) per year. The water in the other stream contains 10 grams/MCM of an industrial pollutant. This stream flows into the lake at the rate of 0.3 MCM per year. There is also an outlet from the lake which keeps the volume of the lake constant at 50 MCM. The water in the lake currently contains 2 grams/MCM of the pollutant. Assuming that the industrial polluter has been releasing the pollutant at a constant rate, how many years ago did the pollution of the lake start? Give a decimal answer.