Your Name (PRINT clearly)

Your Student ID Number

CIRCLE your Professor's Name:

Prof. Camacho
Prof. Bube

CIRCLE your Lecture Section:

A (9:30) or B (10:30)
C (hybrid)

Read these INSTRUCTIONS:

- Turn off and stow away all cell phones and any other electronic or connected devices.
- This exam is closed book. You may use one (two-sided) $8.5'' \times 11''$ sheet of handwritten notes. Do not share notes.
- You may only use a TI-30X IIS calculator model. No other calculator models are allowed.
- In order to receive credit, you must **show clear and complete work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place your final answers in the indicated areas. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but **if you want us to grade work written on these spare pages, you must clearly indicate in the problem area** that your work is on the back of the cover page or on the blank page at the end of the exam.
- This exam has 9 problems on 9 pages. **When the exam starts, make sure that your exam is complete.**

Good luck!

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

1. Evaluate the following integrals. Show clear and complete work. Simplify your answers.

(a) (6 points) $\int x^3 \ln(x)\, dx$

Answer: _____

(b) (7 points) $\int \frac{x}{\sqrt{x^2+2x+5}}\, dx$

Answer: _____

2. Evaluate the following integrals. Show clear and complete work. Simplify your answers.

(a) (6 points) $\int_0^1 (1+x^{1/3})^2\,dx$

Answer: _____

(b) (7 points) $\int_1^2 \frac{x^2-2x+3}{x^3+x^2}\,dx$

Answer: _____

3. (10 points) Evaluate the following improper integral. Simplify your answer but leave it in exact form. For credit, you must show all appropriate limits and computations.

$$\int_4^\infty \frac{3}{\sqrt{x}(4+x)}dx$$

Answer: _____

4. Let $f(x)$ be a continuous function defined for all $x \geq 0$.

   You are given the following information about f:

$$\begin{aligned} \int_0^1 f(x)\,dx &= 2, \\ \int_1^2 f(x)\,dx &= 3, \\ \int_1^2 xf(x^2)\,dx &= 7, \\ \int_1^2 x^2 f(x^3)\,dx &= 10. \end{aligned}$$

   (a) (4 points) Find $\displaystyle\int_0^2 f(x)\,dx$.

   Answer: _____

   (b) (3 points) $\displaystyle\int_1^4 f(x)\,dx$.

   Answer: _____

   (c) (3 points) $\displaystyle\int_4^8 f(x)\,dx$.

   Answer: _____

5. (10 points) You are in a building looking out a window. A tomato is thrown upward from the top of the building. Because of gravity, it eventually reverses direction and starts to fall, passing your window four seconds after being thrown. At the moment when the tomato passes your window, the tomato has a speed of 96 ft/sec. The tomato takes 5 seconds to hit the ground after being thrown.

   Find the height of your window above the ground.

   Assume that gravity is the only force acting on the tomato, and the acceleration due to gravity is 32 ft/sec^2. Use Calculus and show your work.

Answer: Height of window = _____ feet

6. (10 points) A spherical tank of radius 2 meters is buried in the ground.

   The top of the tank is 1 meter under the ground level, and a pipe connects the top of the tank to the ground level. The tank is full of water (density 1000 kg/m^3).

   Find the work (in Joules) required to pump all of the water out of the tank through the pipe, up to the ground level. Use $g = 9.8$ m/sec^2 for the acceleration due to gravity.

Answer: work= _____ Joules

7. (12 points) Consider the region R bounded by the curves

$$y = \cos(2x) \text{ and } y = \cos x, \text{ between } 0 \leq x \leq \frac{2\pi}{3}.$$

(a) (4 points) Compute the area of this region.

Answer: Area= _____

(b) (8 points) Compute the center of mass of this region. Give your answer in exact simplified form.

Answer: $(\bar{x}, \bar{y}) =$ _____

8. (10 points) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dt} = 2e^{7y-t}, \quad y(0) = 0.$$

For full credit, show all your steps and write your answer in explicit form $y = f(t)$.

Answer: $y(t) =$ _____

9. (12 points) A thin tissue culture is growing between two thin glass slides of area 100 cm^2. Let $A(t)$ be the area of the culture in cm^2 as a function of time t in hours. Because most cell divisions occur on the periphery of the tissue, and because the culture cannot grow beyond the edges of the slides:

the rate of change of $A(t)$ is proportional to the product $\sqrt{A(t)}$ times $100 - A(t)$.

(a) (3 points) Write down a **differential equation** for $A(t)$. Use k for the constant of proportionality.

Differential equation: _____

(b) (9 points) Initially, the area of the culture is 4 cm^2. After 9 hours, the area of the culture is 25 cm^2. Find the solution of the differential equation satisfying these conditions. Write your solution in explicit form $A(t) =$ a function of t.

Answer: $A(t) =$ _____

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.