Your Name

Your Student ID Number

Professor's Name

Lecture Section (circle one)

A (9:30), B (10:30), C (hybrid)

- Turn off and stow away all cell phones, smart watches, and other electronic or connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 10 pages. **When the exam starts, make sure that your exam is complete.**

Good luck!

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

1. Evaluate the following integrals. Show your work and box your final answer.

   (a) (5 points) $\int \frac{\ln x}{x^2}\,dx$

   (b) (5 points) $\int \frac{8}{(x-2)(x^2+4)}\,dx$

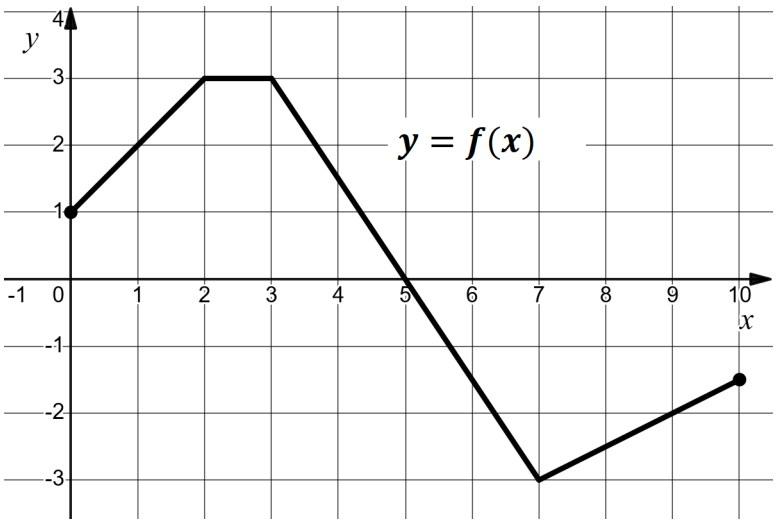
2. Answer the following questions. Show your work and box your final answer.

   (a) (5 points) Evaluate $\int_0^{\sqrt{3}/2} x^2 \sin^{-1}(x)\,dx$

   (b) (5 points) Use the Comparison Test to determine whether the following improper integral converges or diverges. Show your work.

$$\int_3^{\infty} \frac{dx}{x^4 + \cos^2 x}$$

3. (10 points) Let $f(x)$ be the function whose graph, for $0 \le x \le 10$, is shown below:



Let $g(x) = \int_0^x f(t)\,dt$, and let $h(x) = \int_0^{x^2} f(t)\,dt$.

(a) (2 points) Evaluate $g(3)$.

Answer: $g(3) =$ _____

(b) (3 points) Evaluate $g'(3)$.

Answer: $g'(3) =$ _____

(c) (2 points) Evaluate $h(3)$.

Answer: $h(3) =$ _____

(d) (3 points) Evaluate $h'(3)$.

Answer: $h'(3) =$ _____

4. (10 points) A particle is moving along a straight line with acceleration at t seconds, in m/s^2, given by:

$$a(t) = 2t - 6$$

At time $t = 0$ seconds, its velocity is $v(0) = 8$ m/s.

(a) (5 points) Find the velocity $v(t)$ of the particle as a function of time t.

(b) (5 points) What is the *total distance* traveled by the particle from time $t = 0$ to time $t = 4$ sec?

5. (a) (5 points) Find the area between the curve $y = x(1-x)$ and the x-axis over the interval $0 \leq x \leq 1$.

(b) (5 points) Find the constant m, with $0 < m < 1$, such that the area enclosed between the curve $y = x(1-x)$ and the line $y = mx$ is half of the total area from part (a).

6. (10 points) Let R be the region with $x \geq 0$ enclosed between the graphs of

$$y = \frac{9}{x^2} \text{ and } y = 10 - x^2.$$

Set up an integral equal to the volume of the solid of revolution obtained by rotating this region R about the line $x = -1$. Do not compute or simplify the integral, just set it up.

7. (10 points) Compute the *arc length* of the curve $y = e^x$ over the interval $[0, 1]$.

8. (10 points) A bucket of mass 4 kg is initially filled with water of mass 30 kg.

   The bucket and water are lifted from the bottom of a well 10 meters deep up to the top of the well at a constant speed with a rope of mass 0.5 kg per meter.

   The bucket leaks water at a constant rate and only 10 kg of water get to the top of the well.

   How much work, in Joules, is done? You may use $g = 9.8$ m/s^2.

9. (10 points) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dx} = (y^2+1)\sqrt{4-x^2}, \quad y(0) = 1.$$

For full credit, write your answer in explicit form, $y = f(x)$.

10. (10 points) A tank initially contains 90 liters of pure water.

    A solution of salt in water that contains 0.2 kg of salt per liter enters the tank at a rate of 10 liters/min. In addition, another solution of salt in water that contains 0.1 kg of salt per liter enters the tank at a rate of 5 liters/min.

    The solution is kept thoroughly mixed in the tank, and it drains from the tank at a rate of 15 liters/min.

    How much salt will there be in the tank after t minutes?

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