Your Name

Your Signature

Student ID #

Lecture Section (A, B, or C)

Professor's Name

TA's Name (optional)

- Turn off and stow away all cell phones, smart watches, and other electronic or connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place a box around your answer to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, make sure that your exam is complete. Good luck!

1. (10 total points) Evaluate the following integrals.

   (a) (5 points) $\displaystyle\int \frac{2x^4 - 29x^2 + 12x + 1}{x^2 + 4x}\,dx$

   (b) (5 points) $\displaystyle\int \frac{1}{\sqrt{x^2 + 6x + 1}}\,dx$

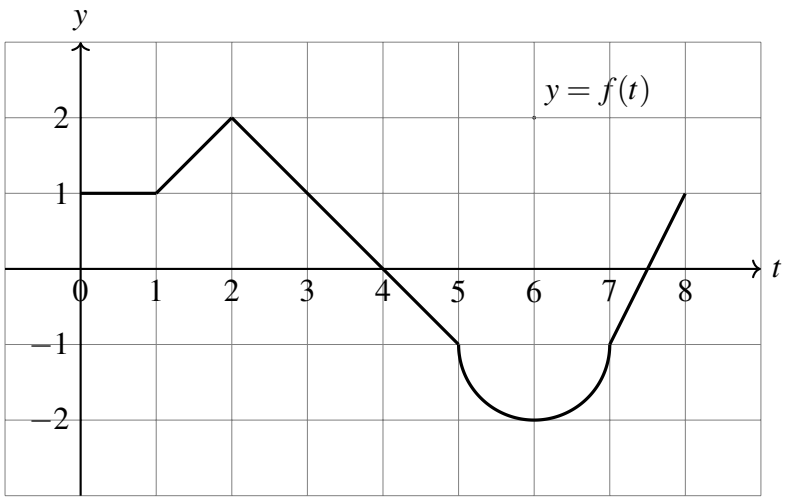
2. (10 total points) Evaluate the following integrals. Show your work.

   (a) (5 points) $\int_0^{\pi} x^2 \sin(3x)\,dx$

   (b) (5 points) $\int_0^3 \frac{\sqrt{x}}{x+9}\,dx$

3. (10 total points) **For this problem, you do not need to show your work.** Answer the following questions based on the graph of $y = f(t)$ below, which has the domain $0 \le t \le 8$.

   The function $y = f(t)$ consists of a horizontal line segment for $0 \le t \le 1$; two line segments between $1 < x \le 5$; a lower semicircle of radius 1 between $5 < x < 7$; and a line segment between $7 \le x \le 8$.



   (a) (2 points) $\displaystyle\int_0^5 f(t)\,dt =$ _____.

   (b) (3 points) If $\displaystyle g(x) = \int_1^{3x} f(t)\,dt$, then $g'(2) =$ _____.

   (c) (2 points) The average value of $y = f(t)$ over the interval $[6, 8]$ is _____.

   (d) (3 points) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f\left(5 + \frac{2i}{n}\right) \cdot \left(\frac{2}{n}\right) =$ _____.

4. (10 points) You are in a building, at a window. A tomato is thrown downward from the top of the building, passing your window two seconds after being thrown. At the moment when the tomato passes your window, the tomato has a speed of 20 m/sec. The tomato takes five seconds to hit the ground after being thrown. Find the height of your window above the ground.

   Assume gravity is the only force acting on the tomato, with the constant of acceleration being 9.8 m/sec^2.

5. (10 points) Let R be the region in the first quadrant of the xy-plane

$$\text{under the graph of } y = x^{1/3} \text{ and between } x = 0 \text{ and } x = 8.$$

Find the number b such that line $y = bx$ divides the region R in half.

6. (10 total points) Consider the region $\mathcal{R}$ in the xy-plane completely enclosed by the curves $y = 4x - x^2$ and $y = x$.

   (a) (5 points) Set up, but do not evaluate, an integral that computes the volume of the solid of revolution generated by revolving $\mathcal{R}$ about the vertical line $x = -1$.

   (b) (5 points) Set up, but do not evaluate, an integral that computes the volume of the solid of revolution generated by revolving $\mathcal{R}$ about the horizontal line $y = 4$.

7. (10 points) Let R be the region enclosed by

$$y = \frac{1}{x^3},\ y = -\frac{1}{x^3},\ x = 1,\ \text{and}\ x = a > 1.$$

(a) (8 points) Find the the x-coordinate for the centroid (center of mass) of R.
You may assume the y-coordinate of the centroid is zero due to the symmetry of the region.

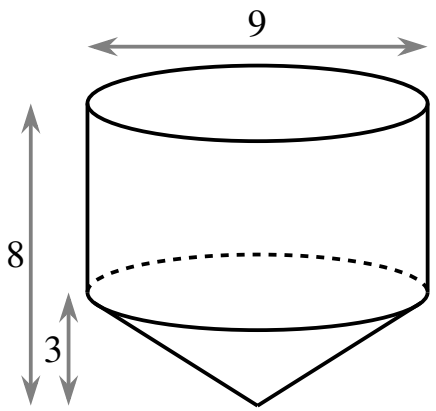
(b) (2 points) What happens to the centroid as a goes to infinity?

8. (10 points)

A tank has the shape of a cylinder on top of an inverted cone. The tank's dimensions, in meters, are shown in the figure.

The tank is full of water, the density of which is 1000 kg/m^3. Assume the acceleration due to gravity is 9.8 m/s^2.

Determine the amount of work done in pumping all of the water to the top of the tank. Give your answer in decimal form, with at least three digits of accuracy.



9. (10 points) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{e^{x+y}}{1+e^x},\ y(0) = -2.$$

Your answer should give y as a function of x.

10. (10 total points)

A tank initially contains 100 liters of water in which 10 grams of salt were dissolved.

Pure water is flowing into the tank at a rate of 2 liters per second.

The liquid in the tank is kept thoroughly mixed, and the the mixed liquid is flowing out of the tank at a rate of 3.5 liters per second.

(a) (2 points) Write down an expression $V = V(t)$ for the volume of mixed liquid in the tank after t seconds.

(b) (3 points) Set up a differential equation for the rate of change $\frac{dy}{dt}$ of the amount y of salt in the tank.

(c) (5 points) How much salt, in grams, is in the tank after 30 seconds? Express your answer as a decimal number with at least three digits of accuracy.