1. (12 points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int \frac{11x-12}{x^3-4x^2+4x}\,dx$

$$\frac{11x-12}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\Rightarrow \quad 11x-12 = A(x-2)^2 + Bx(x-2) + Cx$$

$$11x-12 = (A+B)x^2 + (-4A-2B+C)x + 4A$$

$$4A = -12 \Rightarrow A = -3$$

$$A+B = 0 \Rightarrow B = -A = 3$$

$$-4A-2B+C = 11 \Rightarrow 12-6+C = 11 \Rightarrow C = 5$$

$$\int \frac{11x-12}{x(x-2)^2}\,dx = \int \frac{-3}{x} + \frac{3}{x-2} + \frac{5}{(x-2)^2}\,dx$$

$$= \boxed{-3\ln|x| + 3\ln|x-2| - \frac{5}{x-2} + C}$$

$$= \boxed{3\ln\left|\frac{x-2}{x}\right| - \frac{5}{x-2} + C}$$

   (b) (6 points) $\displaystyle\int \frac{\tan^2(t)\sec^2(t)}{1+\tan(t)}\,dt$

$u = 1+\tan t$

$du = \sec^2 t\,dt$

$$\int \frac{(u-1)^2}{u}\,du$$

$$= \int \frac{u^2-2u+1}{u}\,du$$

$$= \int u - 2 + \frac{1}{u}\,du$$

$$= \frac{1}{2}u^2 - 2u + \ln|u| + C$$

$$= \boxed{\frac{1}{2}(1+\tan t)^2 - 2(1+\tan t) + \ln|1+\tan t| + C}$$

OR

$u = \tan t$

$du = \sec^2 t\,dt$

$$\int \frac{u^2}{1+u}\,du$$

$$\begin{array}{r} u-1 \\ u+1\,\overline{)\,u^2} \\ -(u^2+u) \\ \hline -u \\ -(-u-1) \\ \hline 1. \end{array}$$

$$= \int u - 1 + \frac{1}{u+1}\,du$$

$$= \frac{1}{2}u^2 - u + \ln|u+1| + C$$

$$= \boxed{\frac{1}{2}\tan^2 t - \tan t + \ln|\tan t + 1| + C}$$

(there are other equivalent answers possible)

2. (12 points)

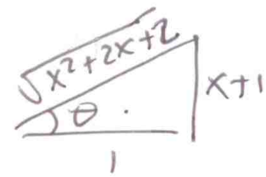
(a) (8 points) Evaluate the indefinite integral: $\displaystyle\int \frac{1}{(x^2+2x+2)^{3/2}}\,dx$

$$= \int \frac{1}{((x+1)^2+1)^{3/2}}\,dx \qquad \boxed{\begin{matrix} x+1 = \tan\theta \\ dx = \sec^2\theta\, d\theta \end{matrix}}$$

$$= \int \frac{1}{(\sec^2\theta)^{3/2}} \sec^2\theta\, d\theta$$

$$= \int \frac{1}{\sec^3\theta} \sec^2\theta\, d\theta = \int \frac{1}{\sec\theta}\, d\theta$$

$$= \int \cos\theta\, d\theta = \sin\theta + C$$



$$= \boxed{\frac{x+1}{\sqrt{x^2+2x+2}} + C}$$

(b) (4 points) Use your answer from part (a) to determine if the following improper integral is convergent or divergent. If convergent, evaluate it. Show all limit computations.

$$\int_0^\infty \frac{1}{(x^2+2x+2)^{3/2}}\,dx$$

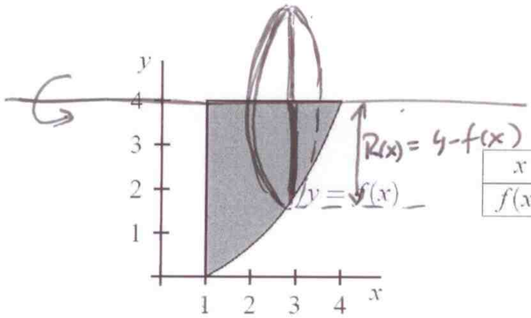
$$= \lim_{t\to\infty} \int_0^t \frac{1}{(x^2+2x+2)^{3/2}}\,dx = \lim_{t\to\infty}\left(\frac{t+1}{\sqrt{t^2+2t+2}} - \frac{0+1}{\sqrt{0^2+0+2}}\right)$$

(dividing by t top & bottom)

$$= \underbrace{\lim_{t\to\infty}\left(\frac{1+1/t}{\sqrt{1+2/t+2/t^2}}\right)} - \frac{1}{\sqrt{2}}$$

$$= 1 - \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}-1}{\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}}$$

3. (10 points) Consider the region between the curve $y = f(x)$, the line $x = 1$, and the line $y = 4$. A formula for $f(x)$ is not known; however, we do have the following picture of the region and a table of values:



x	1	1.5	2	2.5	3	3.5	4
$f(x)$	0	.3	.7	1.2	1.9	2.8	4

(a) (6 points) Use Simpson's Rule with $n = 6$ subintervals to estimate the volume of the solid of revolution obtained by rotating this region around the **horizontal** line $y = 4$.

First, setting up the volume integral, using Disks:

$$V = \int_1^4 \pi R^2(x)\,dx = \int_1^4 \pi\,(4 - f(x))^2\,dx \qquad \left(\Delta x = \frac{4-1}{6} = 0.5\right)$$

$$S_6 = \pi \frac{0.5}{3}\Big[(4-0)^2 + 4(4-0.3)^2 + 2(4-0.7)^2 + 4(4-1.2)^2 + 2(4-1.9)^2 + 4(4-2.8)^2 + (4-4)^2\Big]$$

$$= \frac{\pi}{6}[138.48] = \boxed{23.08\pi} \cong \boxed{72.50796} \text{ cubic units}$$

(b) (4 points) Define $A(x) = \int_1^{1+1/x} f(t)\,dt$, where f is the same function as in the table above. Compute $A'(2)$.

$$A'(x) = \frac{d}{dx}\int_1^{1+1/x} f(t)\,dt = f\left(1+\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$$ by FTC I and Chain Rule

$$A'(2) = \underbrace{f\left(1+\frac{1}{2}\right)}\left(-\frac{1}{2^2}\right)$$

$$= 0.3\left(-\frac{1}{4}\right)$$

$$= \boxed{\frac{-0.3}{4}} = \boxed{-0.075}$$

4. (12 points) The velocity of a particle moving along the number line is given at all times $t \geq 1$ by

$$v(t) = \frac{8}{t^3} - \frac{4}{t^2} \text{ ft/sec.}$$

At time $t = 2$ seconds, the particle's position is $s(2) = 10$ feet.

(a) (6 points) Find the function, $s(t)$, for the position of the particle at time t seconds, $t \geq 1$.

$$S(t) = \int 8t^{-3} - 4t^{-2}\,dt = \frac{8}{-2}t^{-2} - \frac{4}{-1}t^{-1} + C$$

$$= -\frac{4}{t^2} + \frac{4}{t} + C$$

$$S(2) = 10 \Rightarrow -\frac{4}{(2)^2} + \frac{4}{(2)} + C = 10 \Rightarrow C = 9$$

$$\boxed{S(t) = -\frac{4}{t^2} + \frac{4}{t} + 9}$$

(b) (6 points) Find the **total distance** traveled by the particle from $t = 1$ to $t = 4$ seconds.

$$\text{Total distance} = \int_1^4 |v(t)|\,dt = \int_1^4 \left| \frac{8}{t^3} - \frac{4}{t^2} \right| dt$$

$$\frac{8}{t^3} - \frac{4}{t^2} = 0 \iff 8 - 4t = 0 \iff t = 2$$

$$\int_1^2 \frac{8}{t^3} - \frac{4}{t^2}\,dt = \left(-\frac{4}{t^2} + \frac{4}{t}\right)\Big|_1^2 = (-1+2) - (-4+4) = 1 \text{ ft}$$

$$\int_2^4 \frac{8}{t^3} - \frac{4}{t^2}\,dt = \left(-\frac{4}{t^2} + \frac{4}{t}\right)\Big|_2^4 = \left(-\frac{4}{16} + 1\right) - (-1+2) = -\frac{1}{4} \text{ ft}$$

Particle traveled 1 ft in the positive direction, then $\frac{1}{4}$ ft in the negative direction.

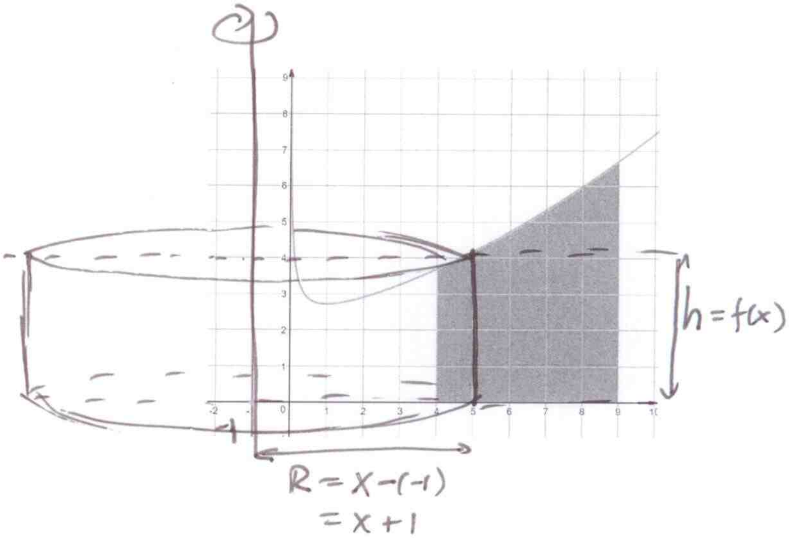
$$\text{Total distance} = 1 + \frac{1}{4} = \boxed{\frac{5}{4} \text{ ft}}$$

5. (12 points) Find the volume of the body of revolution obtained by rotating the region below $y = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$ and above the x-axis, between $x = 4$ and $x = 9$, about the **vertical** line $x = -1$.

SHELLS:

$$V = \int_4^9 2\pi\, r(x)\, h(x)\, dx$$

$$= \int_4^9 2\pi\,(x+1)\,\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$



$u = \sqrt{x} \Rightarrow u^2 = x$
$2u\,du = dx$

$$= \int_2^3 2\pi\,(u^2+1)\,\frac{e^u}{\cancel{u}}\,2\cancel{u}\,du$$

$$= 4\pi \int_2^3 (u^2+1)\,e^u\,du$$

IBP: $w = u^2+1$, $dv = e^u\,du$; $dw = 2u\,du$, $v = e^u$

$$= 4\pi\left[(u^2+1)e^u\Big|_2^3 - \int_2^3 2u e^u\,du\right]$$

IBP: $w = 2u$, $dv = e^u\,du$; $dw = 2\,du$, $v = e^u$

$$= 4\pi\left[10e^3 - 5e^2 - 2ue^u\Big|_2^3 + \int_2^3 2e^u\,du\right]$$

$$= 4\pi\left[10e^3 - 5e^2 - 6e^3 + 4e^2 + 2e^u\Big|_2^3\right]$$

$$= 4\pi\left[4e^3 - e^2 + 2e^3 - 2e^2\right]$$

$$= 4\pi\left[6e^3 - 3e^2\right]$$

$$= \boxed{12\pi e^2\,[2e-1]}$$