• This exam is closed book. You may use one 8.5” × 11” sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.

• No calculators of any kind are allowed.

• In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

• You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.

• Place a box around your answer to each question.

• If you need more room, use the backs of the pages and indicate that you have done so.

• Raise your hand if you have a question.

• This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.
1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) \( \int \frac{x}{\sqrt{x+2}} \, dx \)

(b) (5 points) \( \int e^{2x} \sec(e^{2x}) \tan^3(e^{2x}) \, dx \)
2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) \( \int_{1}^{2} \frac{\ln x}{x^3} \, dx \)

(b) (5 points) \( \int_{2}^{3} \sqrt{4x - x^2} \, dx \)
3. (10 points) Find the area under the curve

\[ y = \frac{1}{\sqrt{|2x - x^2|}} \]

and above the x-axis, for \(-1 \leq x \leq 1\). **CAUTION:** The integral you will get is an improper integral. Be sure to treat your integral as an improper integral, and justify your answer.
4. (10 points) Let $A(t)$ denote the area under the curve $y = \sqrt{1-x^3}$ and above the $x$-axis, between the vertical lines $x = t$ and $x = 2t$.

Find the value of $t$ for which $A(t)$ is a maximum on the interval $0 \leq t \leq 1/2$.

Justify that your answer gives the maximum.
5. (10 points) The region under the curve $y = 2x - x^2$ and above the $x$-axis is rotated around the line $x = -b$, where $b$ is a positive constant. Find the value of $b$ for which the volume of the solid so obtained is $10\pi$. 
6. (10 points) An 80-ft cable is used to lift 50 pounds of coal up a mine shaft 80 ft deep. The bottom half of the cable weighs 2 pounds per foot and the top half of the cable weighs 3 pounds per foot. Find the work done in foot-pounds.
7. (10 total points) This problem gives one way to find a rational number that approximates \( \pi \).

(a) (4 points) Show that
\[
\int_{0}^{4} \frac{dy}{1 + (y^2/16)} = \pi.
\]

(b) (6 points) Subdivide \([0,4]\) into 4 equal subintervals and use Simpson’s rule to approximate the integral in part (a). You do NOT have to simplify any expressions involving fractions.

(For example, if you have terms that look something like \( \frac{2}{1 + \frac{81}{16}} \), just leave them in that form.)
8. (10 points) Let $R$ be the region above the $x$-axis, below the graph of $y = \frac{1}{(x+1)(x+2)}$, between $x = 1$ and $x = 2$. Find the $x$-coordinate of the centroid (center of mass) of the region $R$. 
9. (10 points) Find the solution of the initial value problem

\[
\frac{dy}{dt} = \frac{1}{y e^{t/2}}, \quad y(0) = -1.
\]

Give your answer in the form \( y = f(t) \).
10. (10 points) A tank initially contains 15 liters of pure water. Sea water with a salt concentration of 35 grams per liter is added at a rate of 2 liters per minute. In addition, pure water is added at a rate of 1 liter per minute. The solution is kept thoroughly mixed and is drained from the tank at a rate of 3 liters per minute. How much salt is in the tank after $t$ minutes?