Math 125 **Final Examination** **Spring 2012**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is allowed, but graphing and/or programmable calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 495 of the text (p. 484 if you have the 6th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	8	
4	10	
5	10	

Question	Points	Score
6	8	
7	8	
8	10	
9	10	
10	12	
Total	100	

1. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int \frac{x^2+1}{\sqrt{1-9x^2}}\,dx$

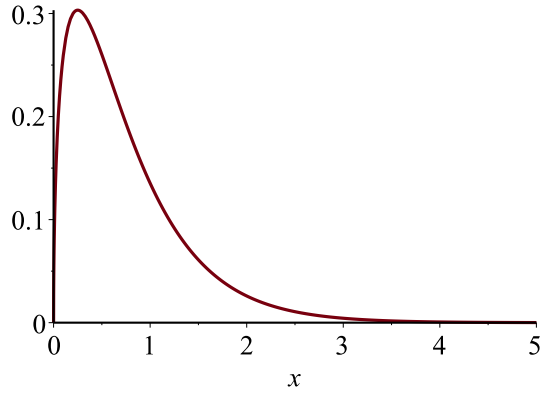
   (b) (6 points) $\displaystyle\int_{-1}^{2} x^5\sqrt{8-x^3}\,dx$

2. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int_{\pi^2/9}^{\pi^2} \frac{|\cos(\sqrt{x})|}{\sqrt{x}}\, dx$ Give your answer in exact form.

   (b) (6 points) $\displaystyle\int \cos^3 x + \left(\tan^2 x\right)\left(\sec^4 x\right)\, dx$

3. (8 points) The infinite region bounded by the curve $y = \sqrt{x}e^{-x}$ and the x-axis is rotated about the x-axis to form a solid of revolution. Does this solid have a finite volume? If so, calculate the volume of the solid.



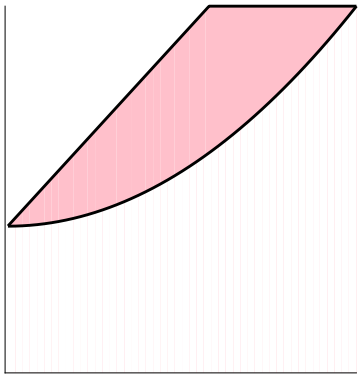
4. (10 total points) A rocket is taking off, going straight up. At time t (in minutes) its height $s(t)$ (in miles) is given by the formula $s(t) = \int_0^{2\pi t/3} \sqrt{4\sin^2 x + 1}\, dx$.

   (a) (5 points) Estimate the height $s(1) = \int_0^{2\pi/3} \sqrt{4\sin^2 x + 1}\, dx$ of the rocket at time $t = 1$ using Simpson's rule with $n = 4$ equal subintervals. Show your work carefully, and give us enough information so we know what you are entering into you calculator. Give your answer (in miles) in decimal form with at least 2 digits after the decimal point.

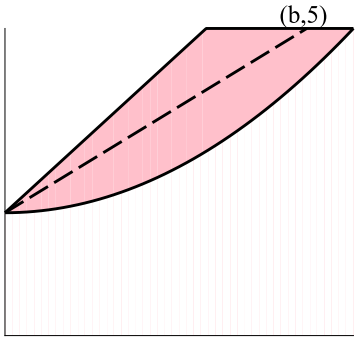
   (b) (5 points) At time $t = 1$, the engine on the rocket shuts off, and the only force on the rocket comes from the acceleration of gravity (21.8 miles/min^2). At what time t (in minutes) will the rocket hit the ground?

5. (10 total points) Let $\mathscr{R}$ be the region in the first quadrant bounded by the curves $y = 2 + x^2$ on the right, $y = 5$ on top, and $y = 3x + 2$ on the left.

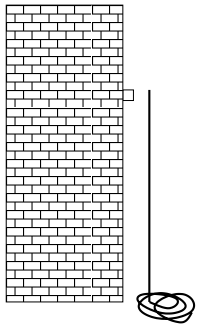
   (a) (5 points) Find the area of the region $\mathscr{R}$.



   (b) (5 points) The line through $(0,2)$ and $(b,5)$ divides $\mathscr{R}$ into two regions of equal area. Find b.

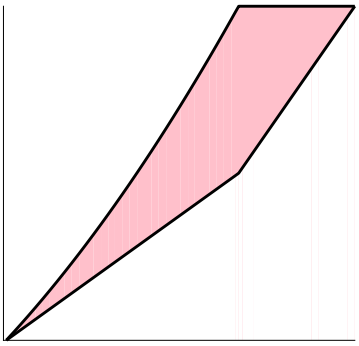


6. (8 points)



A 50-foot rope weighs 2 pounds per foot. One end of it has been lifted to a window 15 feet above the ground and the rest is lying coiled on the ground (see the picture). What is the work needed to pull the whole rope through the window?

7. (8 points) Let $\mathscr{R}$ be the region which is bounded on the left by the curve $y = \dfrac{x^2}{4} + \dfrac{3x}{2}$, bounded on the right by the lines $y = x$ and $y = 2x - 2$, and bounded above by the line $y = 4$, as shown in the picture below. Find the volume of the solid obtained by rotating $\mathscr{R}$ about the y-axis.



8. (10 points) Find the centroid $(\bar{x},\bar{y})$ of the region between the curve $y = 3\cos x$ and the x-axis for $0 \leq x \leq \pi/2$.

9. (10 points) Find the solution to the initial value problem

$$\frac{dy}{dx} = 5y^2\left(1 - \frac{3}{y}\right), \qquad y(1) = 4.$$

Give your answer in the form $y = f(x)$.

10. (12 total points) Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. The cooling constant, which is the proportionality constant in the differential equation, remains constant throughout this question.

    (a) (8 points) Experimenting, you have found that 12 ounces of 180°F coffee in your favorite cup will take 20 minutes to cool to a drinking temperature of 115°F in a 70°F room. Set up a diffential equation for the temperature of the coffee, solve it, and find the cooling constant.

    (b) (4 points) One day in the same 70°F room, after having waited 5 minutes for your 12 ounces of 180°F coffee in your favorite cup to cool, you pour in 2 ounces of 32°F water. Assume that when you add the water to the coffee, the two liquids are mixed instantly, and the temperature of the mixture instantly becomes the weighted average of the temperature of the coffee and of the water (weighted by the number of ounces of each fluid). How long will it take (after the initial time when the coffee was 180°F) for the mixture to reach a drinking temperature of 115°F? Give your answer (in minutes) in decimal form with at least 2 digits after the decimal point.