Math 125 **Final Examination** **Spring 2011**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is allowed, but graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 484 of the text (p. 506 if you have the 5th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	10	
4	8	
5	10	

Question	Points	Score
6	10	
7	10	
8	8	
9	10	
10	10	
Total	100	

1. (12 total points) Evaluate the following indefinite integrals.

   (a) (6 points) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$ (for $x \geq 3$)

   (b) (6 points) $\displaystyle\int \frac{1}{(x+3)(\sqrt{x+4}-2)}\,dx$

2. (12 total points) Evaluate the following definite integrals.

   (a) (6 points) $\int_0^{\pi/2} \left(\sin^2\theta + \cos\theta\right) \sin^3\theta \, d\theta$ Give your answer in exact form.

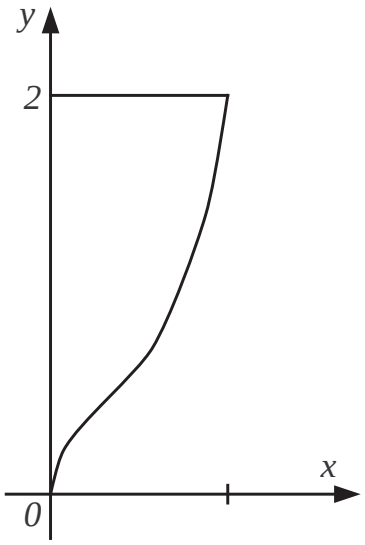
   (b) (6 points) $\int_0^2 t^3 e^{t^2-9} \, dt$ Give your answer in exact form.

3. (10 points) Consider the improper integral $\int_1^\infty \frac{\tan^{-1}(x)}{x^2}\,dx$.

   Evaluate the integral (if it converges) or explain carefully why it does not converge.
   If it converges, give your answer in exact form.

4. (8 points) Let $h(x) = \displaystyle\int_{\cos x}^{\ln x} \frac{e^{2t}}{\sqrt{t^2+1}}\, dt$. Find the derivative $h'(x)$.

5. (10 points) Find the area of the region in the first quadrant bounded on top by the line $y = 2$, on the left by the y-axis, and on the bottom by the curve $y = \sqrt{x/(1-x)}$. Give your answer in exact form.



6. (10 points) Let $\mathscr{R}$ be the region bounded on top by the curve

$$y = 5 + \frac{1}{1+x^2},$$

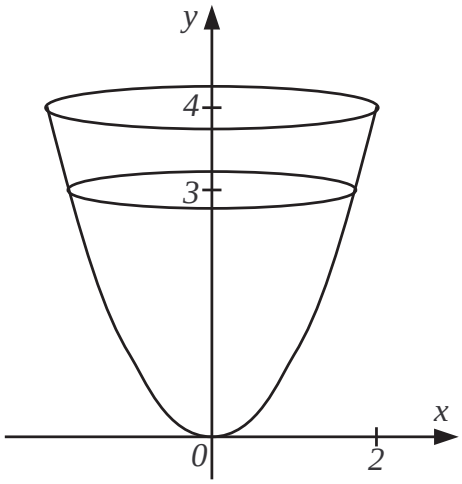
on the bottom by the x-axis, on the left by the line $x = 2$, and on the right by the line $x = 3$.
Find the volume of the solid obtained by rotating $\mathscr{R}$ about the vertical line $x = -2$.
Give your answer in exact form.

7. (10 points) The portion of the graph $y = x^2$ between $x = 0$ and $x = 2$ is rotated around the y-axis to form a container. The container is partially filled with water, up to the level $y = 3$.

   Find the work required to pump all of the water out over the top of the side of the container.

   Give your answer (in joules = $\text{kg} \cdot \text{m}/\text{s}^2 \cdot \text{m}$) in exact form.

   (Distance is measured in meters, the density of water is 1000 kg/m^3, and use 9.8 m/s^2 for the acceleration g due to gravity.)



8. (8 total points) Consider the curve $y = x + \frac{2}{3}x^{3/2}$.

   (a) (4 points) Set up a definite integral for the arc length of this curve for $1 \leq x \leq 3$.
   DO NOT EVALUATE THE INTEGRAL.

   (b) (4 points) Use Simpson's rule with $n = 4$ subintervals to estimate the integral in part (a).
   Give your answer in decimal form, correct to at least the third digit after the decimal point.

9. (10 points) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{2\sqrt{4-y^2}}{ye^x}$$

that satisfies the condition $y(0) = \sqrt{3}$. Solve for y, giving your answer in the form $y = f(x)$.

10. (10 total points) An object of mass m kg is dropped out of a airplane, and we assume that air resistance is proportional to the speed of the object. Let $s(t)$ be the distance dropped (in meters, positive pointing *down*) after t seconds, and let $v(t) = ds/dt$ be the velocity and $a(t) = dv/dt$ be the acceleration. The combined downward force on the object is

$$F = mg - kv,$$

where $g = 9.8$ meters$/\sec^2$ is the acceleration due to gravity and k is a positive constant. By Newton's Second Law of motion,

$$F = ma = m\frac{dv}{dt}.$$

The mass of the object is $m = 10$ kg, and the constant k is $k = 2$.

(a) (3 points) Set up a differential equation for the velocity $v(t)$.

(b) (5 points) Solve the differential equation to obtain a formula for $v(t)$.

(c) (2 points) What is the limiting velocity $\lim_{t\to\infty} v(t)$?