This exam is closed book. You may use one 8.5” × 11” sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.

Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.

A scientific calculator is allowed, but graphing calculators are not allowed.

In order to receive credit, you must show all of your work. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

You may use any of the 20 integrals in the table on p. 484 of the text (p. 506 if you have the 5th edition of Stewart) without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.

Place a box around your answer to each question.

If you need more room, use the backs of the pages and indicate that you have done so.

Raise your hand if you have a question.

This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.
1. (12 total points) Evaluate the following integrals.

(a) (6 points) \[ \int \sec^3(2\theta) \tan^5(2\theta) \, d\theta \]

(b) (6 points) \[ \int \frac{e^x}{e^{2x} + 3e^x + 2} \, dx \]
2. (12 total points)

(a) (6 points) Evaluate the definite integral \( \int_{-2}^{2} (x + 2) \sqrt{4 - x^2} \, dx \)

(b) (6 points) Evaluate the improper integral \( \int_{1}^{2} \frac{1}{\sqrt{x^2 - 1}} \, dx \)
3. (8 total points) Use the area information given on this graph of $f(x)$ to evaluate the integrals below.

(a) (2 points) \[ \int_{3}^{6} |f(x)| \, dx \]

(b) (2 points) \[ \int_{0}^{5} 2 + f(x) \, dx \]

(c) (2 points) \[ \int_{6}^{5} 2f(x) \, dx \]

(d) (2 points) \[ \int_{0}^{3} 6x - f(x) \, dx \]
4. (6 points) The table below shows the velocity of a truck (in ft/sec) as a function of time (in sec) after a stop light turns green.

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
</tbody>
</table>

Use Simpson’s rule with \( n = 6 \) subintervals to approximate the distance the truck travels in the first 6 seconds.
5. (10 total points) Two cars, an Audi and a BMW, start driving in the same direction on a straight road at the same time and from the same place. The following two graphs show the velocities of the two cars (in miles per minute) along that road during the first 12 minutes of the trip. The dashed graph on the left labeled $v_A$ represents the Audi’s velocity, while the solid graph on the right labeled $v_B$ shows the BMW’s velocity. Use the graphs to answer the following questions.

(a) (2 points) What is the total distance traveled by the BMW during the 12 minutes?

(b) (2 points) How far apart are the cars from each other after 4 minutes, and which car is ahead at that time?

(c) (2 points) What is the average velocity of the BMW during the first 4 minutes?

(d) (4 points) Define the function $f(x)$ to be $f(x) = \int_0^{2x} v_B(t) \, dt$. Compute $f'(2)$. 
6. (10 points) A container has the shape of an inverted circular cone with height 10 feet and top radius 4 feet. It is filled with a liquid weighing 60 lb/ft³. Find the work required to pump the top 5 feet of the liquid to the top of the container, and give your answer in decimal form.

*Please label your origin and coordinate axis on the figure.*
7. (10 total points)

(a) (6 points) The region in the first quadrant bounded by the \( x \)-axis, the \( y \)-axis, the line \( x = 2 \), and the graph of \( y = \frac{1}{1 + x^2} \) is rotated around the \( y \)-axis to form a solid of revolution. Find the volume of this solid.

(b) (4 points) The region in the first quadrant bounded by the \( x \)-axis, the line \( x = 4 \), and the graph of \( y = \sqrt{x} \) is rotated around the horizontal line \( y = 3 \) to form a solid of revolution. Express the volume of this solid as a definite integral, but DO NOT EVALUATE THIS INTEGRAL.
8. (10 total points) Consider the region \( R \) bounded by the \( x \)-axis, the line \( x = e \), and the graph of \( y = \ln x \).

(a) (4 points) Find the area of \( R \).

(b) (6 points) Find the \( x \)-coordinate \( x \) of the centroid (center of mass) of \( R \).
9. (10 points) Find the solution of the differential equation that satisfies the given initial condition.

\[
\frac{dy}{dx} = \frac{x \sin(x^2)}{y}, \quad y(0) = -2.
\]
10. (12 total points) A 50-gallon tank initially contains 20 gallons of water in which 10 lbs of salt are dissolved. Pure water enters the tank at a rate of 4 gal/min. Simultaneously, a drain is open at the bottom of the tank, allowing the salt-water solution to leave the tank at a rate of 2 gal/min. The solution is kept thoroughly mixed.

(a) (3 points) Find the volume $V(t)$ (in gallons) of the salt-water solution in the tank at time $t$ (in minutes).

(b) (3 points) Write a differential equation for the amount $y(t)$ (in lbs) of salt in the tank at time $t$.

(c) (3 points) Solve this differential equation and use the initial amount of salt in the tank to find a formula for $y(t)$.

(d) (3 points) What is the amount of salt in the tank at the moment that the tank becomes full?