Math 125 **Final Exam** **Autumn 2025**

Your Name

Your Student ID Number

Professor's Name

Lecture Section (circle one)

A (8:30), B (9:30), C (10:30), D (hybrid)

- Turn off and stow away all cell phones and any other electronic or connected devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct. Please make sure to **write clearly**; if your writing is too faint or too messy, we cannot grade it.
- You may use directly the integral formulas in the table below, without deriving them. **Show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place your final answers in the indicated areas. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All exam pages are double-sided except for this cover page and the last page. You may use the blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank pages at the end of the exam.
- This exam has 9 problems on 9 pages. **When the exam starts, make sure that your exam is complete.**

  Good luck!

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

1. Evaluate the following integrals. Show clear and complete work.

   (a) (6 points) $\int \tan^2(5x)\sec^4(5x)\,dx$

   Answer: ______________________________

   (b) (6 points) $\int_1^e \frac{3\ln(x)}{x\sqrt{4+(\ln x)^2}}dx$

   Answer: ______________________________

2. (6 points) Use the Comparison Test to determine if the following integral converges or diverges.

$$\int_1^\infty \frac{1}{\sqrt{x}+x^2}\,dx$$

This improper integral (check one box):

☐ Converges

☐ Diverges

To indicate the specific comparison you used to reach your conclusion, select one correct and relevant fact from each of the two columns below. No need to justify further.

BECAUSE, for all $x \geq 1$:

☐ $\frac{1}{\sqrt{x}+x^2} < \frac{1}{\sqrt{x}}$

☐ $\frac{1}{\sqrt{x}+x^2} < \frac{1}{x^2}$

☐ $\frac{1}{\sqrt{x}+x^2} > \frac{1}{\sqrt{x}}$

☐ $\frac{1}{\sqrt{x}+x^2} > \frac{1}{x^2}$

AND:

☐ $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ converges

☐ $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ diverges

☐ $\int_1^\infty \frac{1}{x^2}\,dx$ converges

☐ $\int_1^\infty \frac{1}{x^2}\,dx$ diverges

3. (10 points) A car is traveling at a constant velocity of 27 m/s when its driver notices a road block ahead. The driver takes 0.5 seconds to react and start braking, and then brakes with a deceleration

$$a(t) = -3t^2 \ m/s^2,$$

where t is the number of seconds since it started braking, until the car comes to a full stop.

How many meters will the car travel from the first moment when the driver noticed the road block until the car comes to a full stop?

Answer: ____________________ meters

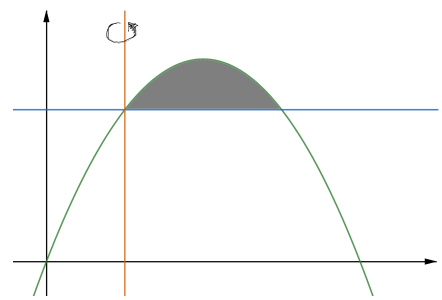
4. (10 points) Let $\mathcal{R}$ be the shaded region shown. It is bounded above by curve:

$$y = 1 + \sqrt{1 - x^2}, \text{ for } 0 \le x \le 1,$$

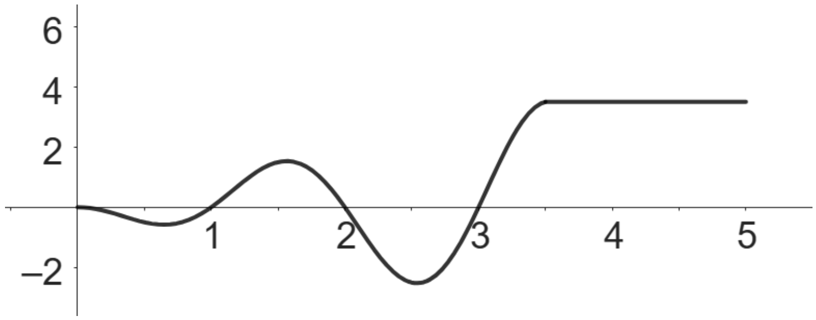
and it is bounded below by the lines $y = 1/2$ and $y = x$.

Set up, but do not evaluate, an integral (or a sum of integrals) equal to the volume of the solid of revolution obtained by revolving $\mathcal{R}$ around **the vertical line** $x = -1$.

Indicate which method you are using, and show some work. Box your final answer.



5. (10 points) Below is shown the graph of a function $y = f(t)$ defined on $0 \leq x \leq 5$.



In addition, you know that: $\int_0^2 f(x)\,dx = 0.6$, $\int_1^2 f(x)\,dx = 0.9$, and $\int_2^4 f(x)\,dx = 1.3$.

(a) Calculate $\int_0^2 |f(x)|\,dx$.

Answer: $\int_0^2 |f(x)|\,dx =$ ______________

(b) Calculate $\int_0^2 xf(x^2)\,dx$.

Answer: $\int_0^2 xf(x^2)\,dx =$ ______________

(c) Let $F(x) = \int_1^x f(t)\,dt$, with f as in the graph above.

i. Circle the correct answer (no need to justify): $F(0)$ is

POSITIVE NEGATIVE ZERO

ii. Circle the correct answer (no need to justify): $F'(1.5)$ is

POSITIVE NEGATIVE ZERO

6. (10 points) The electric force F (in Newtons) acting on a charged particle B as a result of the presence of a second charged particle A is given by Coulomb's Law:

$$F = k\frac{q_A q_B}{r^2},$$

where $k = 9 \times 10^9$ is a constant, q_A and q_B are the charges of the particles A and B, in Coulombs, and r is the distance (in meters) between the particles.

Assume that two particles A and B have charges $q_A = 1$ and $q_B = -1$ Coulomb. (The force acting on them is negative, indicating that the particles are attracting each other.) Assume that particle A is kept fixed, while particle B is being moved away from A by applying a force of equal magnitude and opposite sign to the force by which A attracts B.

(a) Find the work done to move particle B from an initial position of 1 meter away from particle A, to a position 2 meters away from A.

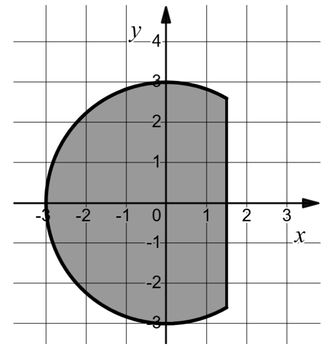
Answer: work= ____________________ Joules

(b) Find the work done to move particle B from an initial position of 1 meter away from A, to an infinite distance away from particle A. If that work is infinite, say so and justify your claim.

Answer: work= ____________________ Joules

7. Let R be the portion of the disk of radius 3 centered at the origin for $-3 \leq x \leq 1.5$, as shown below.

   (a) (8 points) Compute the area of this region R.



AREA = ____________________

   (b) (8 points) Find the coordinates $(\bar{x}, \bar{y})$ for the center of mass of a thin lamina of uniform area density $\rho = 1$ occupying this region R. Round to nearest two decimal digits.

$(\bar{x}, \bar{y})$ = ____________________

8. (10 points) Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dy}{dx} = xe^{2x+y}, \quad y(0) = 1$$

For full credit, show all steps and write your answer in explicit form, $y = f(x)$.

Answer: $y =$ ______________________________

9. A population of yeast cells in certain container is found to satisfy the differential equation:

$$\frac{dP}{dt} = 3P - \frac{P^2}{1000}$$

where $P(t)$ is the number of yeast cells in the container after t hours.

(a) (4 points) What are all the equilibrium solutions ($P =$ a constant) for this differential equation?

Answer: ______________________

(b) (12 points) Solve the given differential equation and determine the number of yeast cells after 2 hours if the population started with 1000 cells. *Show clear and complete work; use additional pages if needed.*

Answer (rounded to nearest whole number of cells) = ____________

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.

Use this blank page for extra space. If you want us to grade it, make sure to state so in the problem area.